Uniport Journal of Engineering and Scientific Research (UJESR)

Vol. 5, Special Issue, 2020, Page 109-119
ISSN: 2616-1192

(www.ujesr.org)

Reliability-Based Safety Assessment of a Castellated Beam

Sule, S*[1] and Bassah, E.J[2]
[1] Department of Civil and Environmental Engineering, University of Port Harcourt, P.M.B 5323, Port Harcourt, Nigeria
[2] Department of Civil Engineering, Federal University Wukari, Taraba State, Nigeria
*Corresponding author's email: samvictoryahead@yahoo.com

Abstract
This paper investigated the safety of a castellated beam with hexagonal openings with respect to the limit state of bending, shear and horizontal shear at web post designed in accordance with the provisions of BS 5950 Part 1, 1985. The safety indices corresponding to each limit state were obtained using a MATLAB program developed based on First Order reliability estimate. The results of the parametric sensitivity analysis with respect to the design parameters showed that the safety indices generally decreased with increase in load ratio and beam span for all the failure criteria considered. The safety indices also increased with increase in area of T-sections, increased as the distance between the centroids of the T-sections increased, decreased as the pitch of castellation increased, increased as the width of web post increased, increased as the bending or yield strength of the beam increased and decreased as the imposed load increased. It was also found that using a beam beyond 16m and a load ratio beyond 1.0 will jeopardize the safety of the beam as they gave negative values of safety indices. The results of the reliability analysis showed that the castellated beam is safe in horizontal shear, safe at some load ratios in shear but unsafe in bending.

Keywords: *Castellated beam, Hexagonal openings, Safety indices, Sensitivity analysis, Limit state*

Received: 18th June, 2020 *Accepted: 9th July, 2020*

1. Introduction

There is an immediate need for structural engineers to improve on the design and construction of structural members. The use of castellated beams as structural members is one of such improvement. A castellated beam is a structural member that has a number of openings in the web region. A castellated beam is formed by flame cutting of a single rolled wide flange beam in a predesigned manner after which the parts are again joined to form a beam with a number of regular openings at the web. The new section with holes now has a depth which is 1.5 times the original depth of section. The formation of a castellated beam increases the stiffness of the steel member without the corresponding increase in the weight of the member. Consequently, the capacity of the beam in flexure, shear and deflection is increased considerably thus making the beam to be used for a longer span (Anupriya and Jagadeesan, 2013). Castellated beams are used in industrial structures and facilities where structural loads are less and the spans are short. According to Mosley and Bungey (2002), structural engineering is not only concerned with the safety and serviceability of structures. It should also consider the intended use of structures. Therefore, the provision of beams with openings at web area has become a suitable engineering practice as it allows for passage of services by a service engineer (Wakchaure and Sagade, 2012). The failure criteria of castellated beams resulting from vierendeel mechanism, lateral torsional buckling of the web, rupture of welded joint in the web, lateral torsional buckling of the overall beam, plastic hinge formation and web buckling have been studied by many researchers (Kerdal and Nethercot, 1984; Redwood and Demirdjian, 1998; Shaikh and Autade, 2016; Amayrey and Saka, 2005). Tkacelvic et al. (2007) carried out a laboratory study on the safety of castellated beams subjected to lateral-torsional buckling using buckling curves a, b, c and d. The safety index obtained from each buckling curve was compared with the target safety index value and they found that only curves b and c met the target safety index of 3.8 recommended for structural members of Safety Class 2 (NKB, 1978). The load effects and structural resistance parameters are uncertain quantities. Consequently, the achievement of absolute safety of structures is not possible. The uncertainties associated with the design parameters can only be quantified using probability and statistics. According to Afolayan (2002), the problems of civil engineering structures are stochastic in nature. Consequently, the application of empirical safety factors in structural engineering design may not guarantee the safety of structures (Melchers, 1999; Sule and Benu, 2019; Abubakar, 2006; Ranganathan, 1999). Probability and statistics have been used to assess the safety of civil engineering structures (Abubakar and Aliyu, 2017; Uche and Afolayan, 2008; Thoft-Christensen and Baker, 1982; El-Reedy, 2013). The increased number of death and damage of properties resulting from structural failure and collapse of steel structures in Nigeria in recent times is a source of concern. A typical example was the collapse of steel girders on Reigners Bible Church that left so many worshippers dead in Akwa Ibom state, Nigeria (The Guardian Newspaper, 10th December, 2016). The failure of the roof structure must have been triggered by underestimation of the uncertain load quantities during structural engineering design.

This paper therefore investigates the safety of a castellated beam with respect to the limit state of bending, shear and horizontal shear at web post, in accordance with the design requirements of BS 5950 Part 1, 1985, is carried out based on First Order Reliability estimate. The limit state functions developed based on the failure criteria were solved by invoking an optimization algorithm which was written in MATLAB language to obtain the safety indices.

2. Development of failure functions

2.1 Design criteria

The failure functions were developed according to the provisions of BS 5950: Part 1, 1985, for the design of steel structures. The castellated beam with hexagonal openings considered in this study is simply supported and subjected to a uniform loading (Fig. 1a).

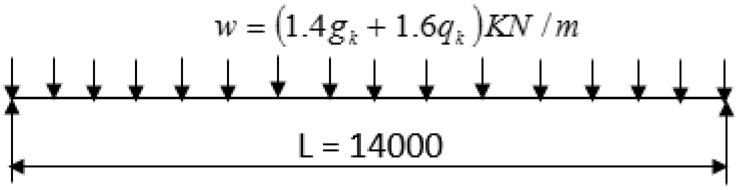

Fig. 1a: A castellated steel beam under uniform loading

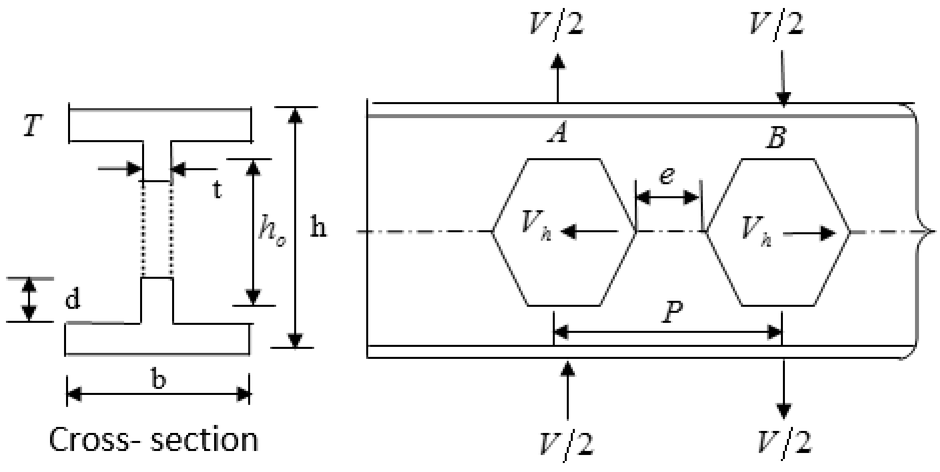

Fig. 1b: Forces on web post of castellated beam

2.2 Bending criterion

The failure condition in bending is an inequality problem given by:

$$M_u \leq M_{app} \quad (1)$$

where M_u = ultimate moment of resistance of the beam, M_{app} = maximum design moment.

The maximum design bending moment is given by:

$$M_{app} = \frac{wL^2}{8} \quad (2)$$

The maximum design bending moment at ultimate limit state is given by:

$$M_{app} = \frac{(1.4a + 1.6)q_k L^2}{8} \quad (3)$$

The moment of resistance of a castellated beam is given by:

$$M_u = A_T P_y h_0 \quad (4)$$

A_T = area of T-section, h_0 = distance between the centroids of T-section, P_y = bending or yield strength of steel, L = beam span.

The limit state function in bending at ultimate limit state is given by:

$$G(X) = A_T P_y h_0 - \frac{(1.4a + 1.6)q_k L^2}{8} \quad (5)$$

$$a = \frac{g_k}{q_k} \quad (6)$$

where a = load ratio.

2.3 Shear stress criterion

The failure condition in shear is given by:

$$\tau_{app} \leq \tau_{perm} \quad (7)$$

where τ_{app} = design shear stress on the web of castellated beam, τ_{perm} = permissible shear stress on the web of a castellated beam.

The design shear stress on the web of castellated beam at ultimate limit state is given by:

$$V = \frac{q_k(1.4a + 1.6)L}{2td} \quad (8)$$

According to BS 5950, Part 1, 1985, the shear strength of the beam is given by:

$$\tau_{all} = 0.70P_y \quad (9)$$

Applying equation (8) and (9), the limit state function in shear is given by:

$$G(X) = 0.70P_y - \frac{q_k(1.4a + 1.6)LP}{2td} \quad (10)$$

where t, d = thickness of stem and depth of stem of T-section respectively.

2.4 Horizontal shear at web post

The horizontal shear force V_h on welded joint in the web post due to change in axial force
in the T-sections is obtained by taking moments at point A or B of Figure 1b.

$$V_h = \frac{VP}{h} \quad (11)$$

According to BS 5950, Part 1, 1985, the shear strength of the web is given by:

$$\tau_{all} = 0.60P_y \quad (12)$$

The maximum horizontal shear stress on the web post at ultimate limit state is given by:

$$\tau_{app} = \frac{q_k(1.4a + 1.6)LP}{2hte} \quad (13)$$

The limit state function due to horizontal shear on the web post is given by:

$$G(X) = 0.60P_y - \frac{q_k(1.4a + 1.6)LP}{2hte} \quad (14)$$

where h, e, P, q_k = overall depth of castellated beam, width of web post, pitch of castellated beam and imposed load on the beam respectively.

3. Materials and methods

3.1 First order reliability method

According to First Order Reliability estimate, the limit state of a structural system is a function of many uncertain variables. The random vector $X = (X_1, X_2, \dots, X_n)^T$ represents uncertain resistance and load quantities that affect the failure scenario, with joint distribution density function given by:

$$F_X(X) = P\left(\bigcap_{i=1}^{n} \{X_i \leq x_i\}\right) \quad (15)$$

where $F_X(X)$ denotes the multivariable density function of X. The safety margin, $G(X)$ of a structure depends on the basic variables associated with a particular limit state. $G(X)$ denotes the safety margin defined such that $G(X) > 0$ represents the safe state of the structure, $G(X) < 0$ represents the unsafe state of the structure and $G(X) = 0$ is the failure surface and it represents the boundary between the safe and unsafe state of the structure.

The probability of failure of a structure can be defined as:

$$p_f = P[G(X) \leq 0] = \phi(-\beta) \quad (16)$$

where β = safety index and it represents the minimum distance from the origin to the failure surface, $G(X) = 0$. It is given by:

$$\beta = \min\{\|x\|\} \text{ for } \{x: G(X) < 0\} \quad (17)$$

The flowchart showing the optimization algorithm is shown in Fig. 2.

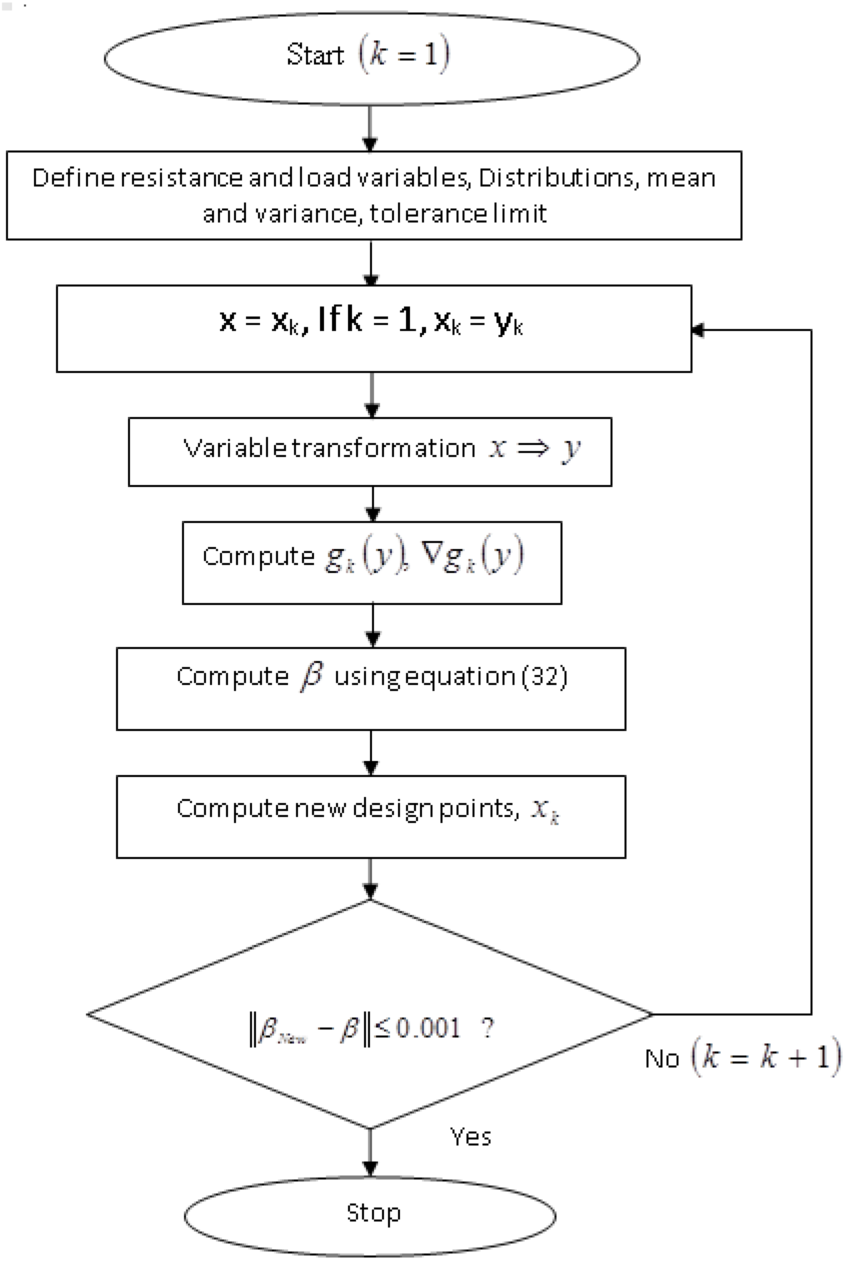

Fig. 2: Flowchart showing the optimization algorithm

3.2 Reliability analysis

A 14m simply supported Grade 43 castellated steel beam of strength class S275 with hexagonal openings at the web area carrying a dead load of 4KN/m and an imposed load of 4KN/m was designed in accordance with the provisions of BS 5950, Part 1, 1985. The probabilistic models of the basic random variables obtained from deterministic analysis are presented in Table 1.

4. Results and discussion

The safety indices for the limit state of bending, horizontal shear and shear considered in this study were obtained using a MATLAB program written based on First-order reliability estimate. Figures 3 – 7 present the results of the safety indices against varied values of the random variables for bending criterion. Figures 8 – 13 present the results of the safety indices against varied values of the random variables for horizontal shear criterion and Figures 14 – 17 present the results of the safety indices against varied values of the random variables for shear criterion. It can be seen from the plots that for bending and shear criterion, the average values of the safety indices fall below the recommended range of the target safety index value (JCSS, 2001). The horizontal shear capacity criterion would give a safe design (Abubakar and Peter, 2012).

Table 1: Probabilistic models of the basic random variables

Basic variable	Mean	Standard deviation	Coeff. of variation	Probability density function
Pitch of castellation, *P*	494 mm	24.7 mm	0.05	Normal
Overall depth of beam, *h*	690.5 mm	34.525 mm	0.05	Normal
Beam span, *L*	14000 mm	980 mm	0.07	Normal
Load ratio, α	1	-	-	Fixed
Imposed load on beam, q_k	5 KN/m	1.25 KN/m	0.25	Gumbel
Width of web post, *e*	114 mm	5.7 mm	0.05	Normal
Design strength of steel, P_y	275 N/mm^2	27.5N/mm^2	0.10	Lognormal
Effective area of cross-section, *A*	7250 mm^2	580 mm^2	0.08	Normal
Depth of stem of T-section, *d*	99.75 mm	4.9875 mm	0.05	Lognormal
Thickness of stem of T-section, *t*	9.6 mm	0.48 mm	0.05	Normal
Distance between the centroids of T-sections, h_o	647 mm	32.35mm	0.05	Normal
Area of T-section, A_r	3582 mm^2	286.56 mm^2	0.08	Normal

Source: (Ranganathan, 1990; Abubakar and Aliyu, 2017; Abubakar, 2006; Abubakar and Peter, 2012)

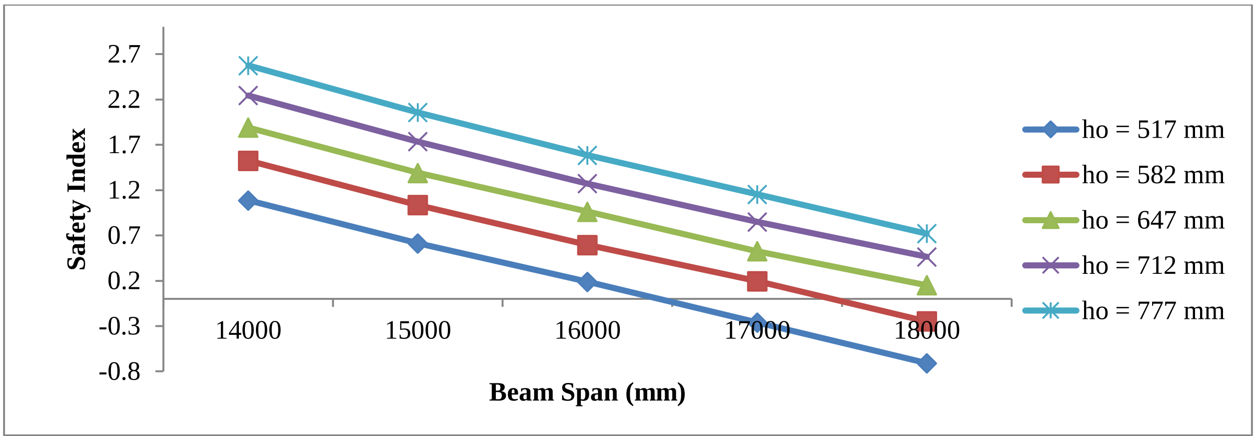

Fig. 3: Safety index against load ratio for varying beam span (Bending criterion)

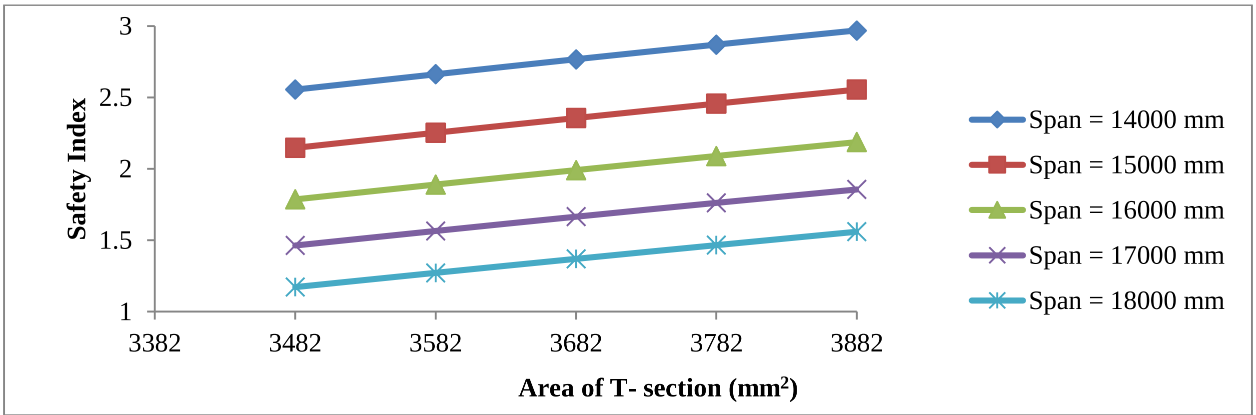

Fig. 4: Safety index against area of Tee-sections for varying beam span (Bending criterion)

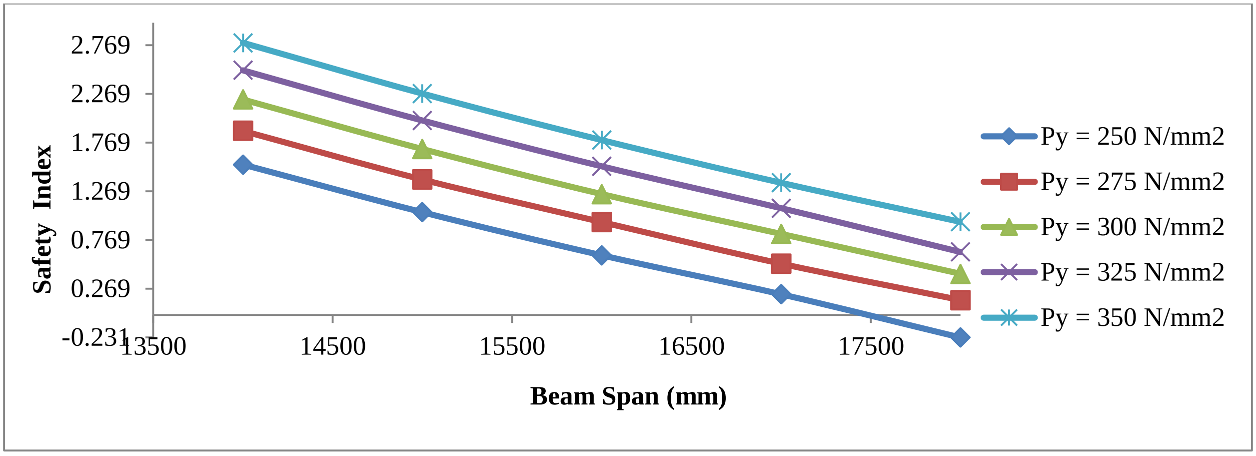

Fig. 5: Safety index against beam span for varying beam strength (Bending criterion)

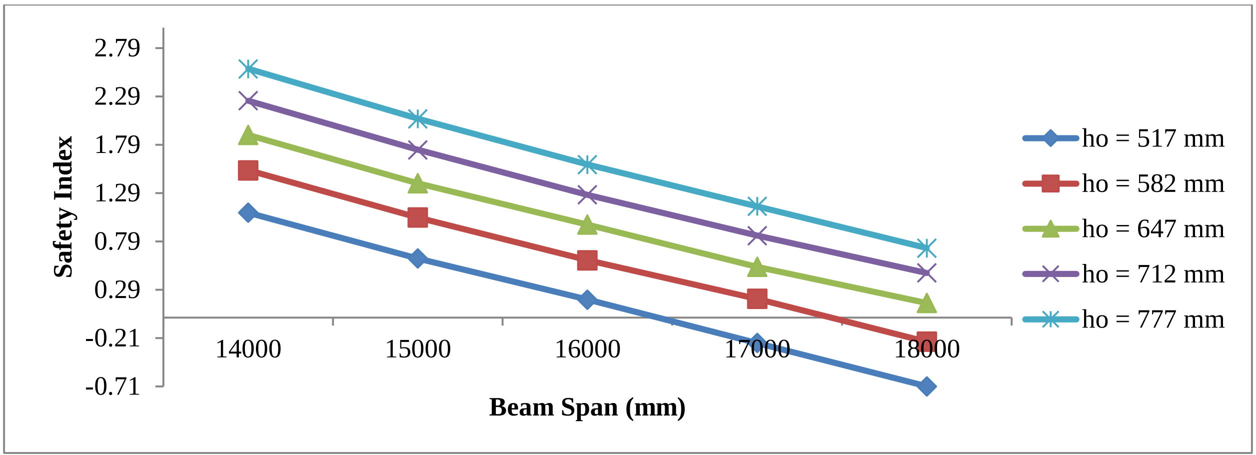

Fig. 6: Safety index against beam span for varying distance between the centroids of T-sections (Bending criterion)

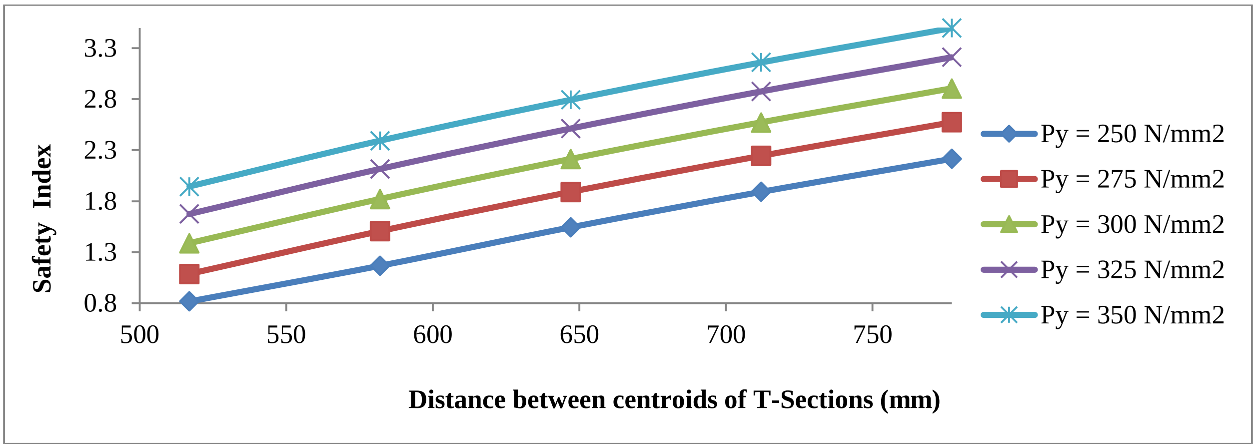

Fig. 7: Safety index against distance between centroids of T-sections for varying bending strength (Bending criterion)

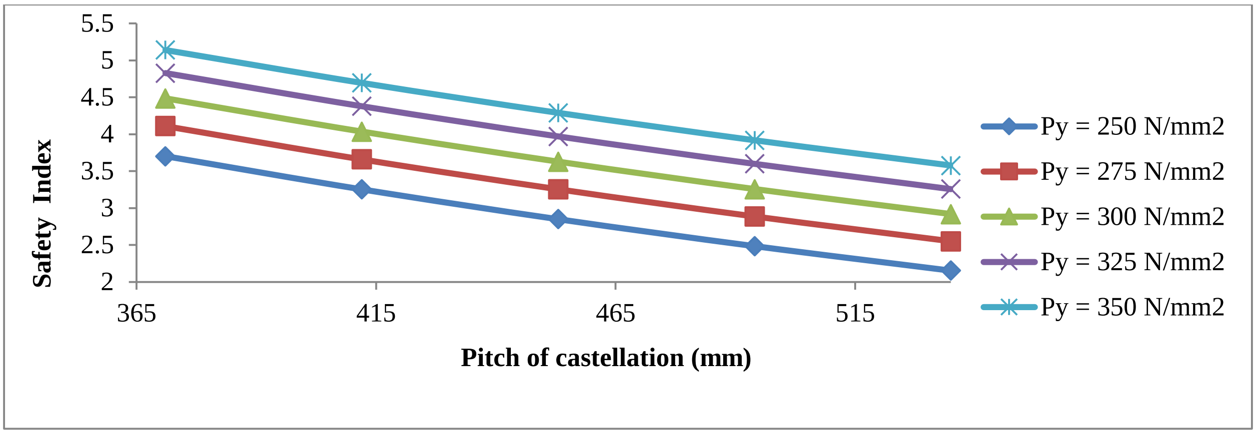

Fig. 8: Safety index against pitch of castellation for varying bending strength (Horizontal shear criterion)

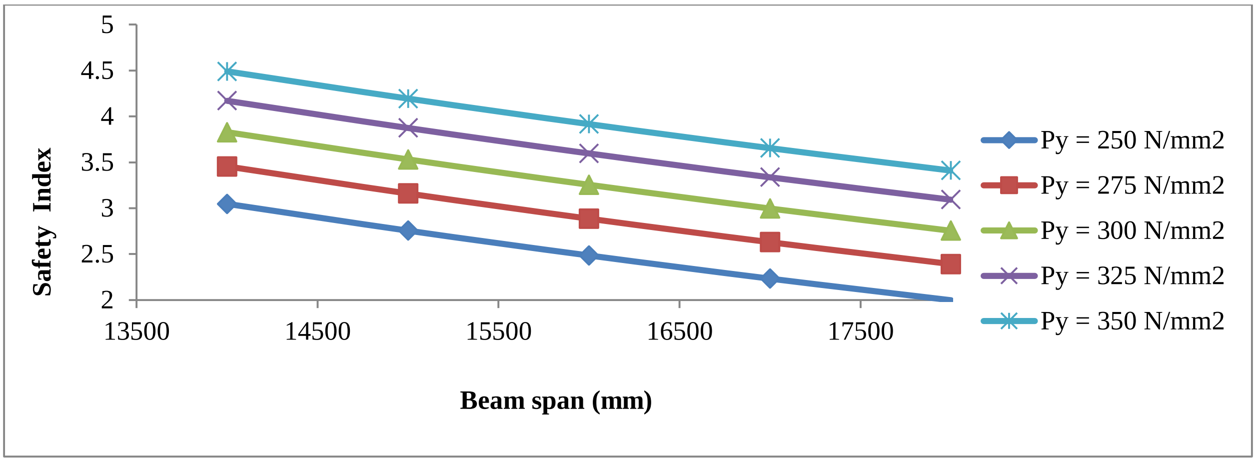

Fig. 9: Safety index against beam span for varying bending strength (Horizontal shear criterion)

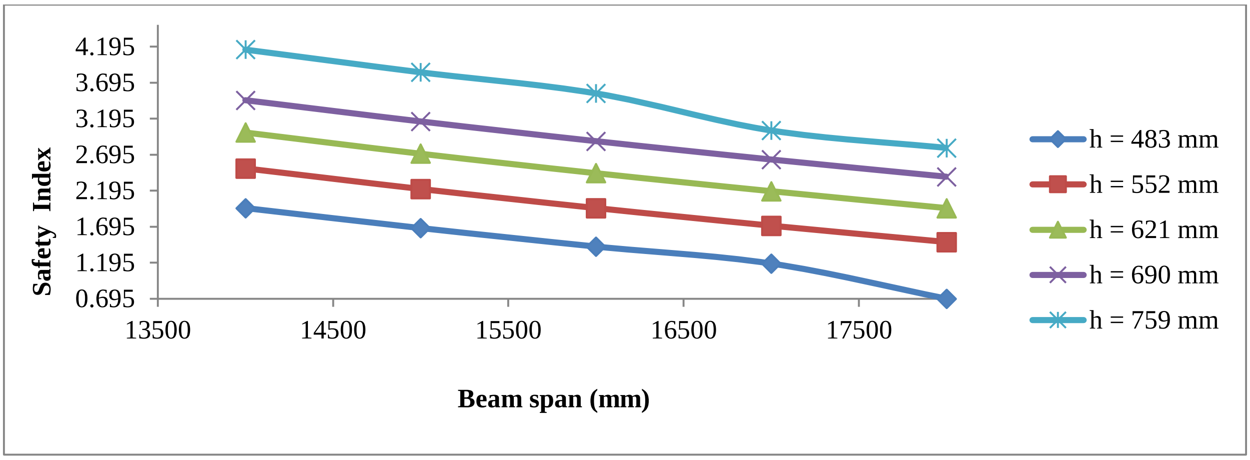

Fig. 10: Safety index against beam span for varying overall depth of beam (Horizontal shear criterion)

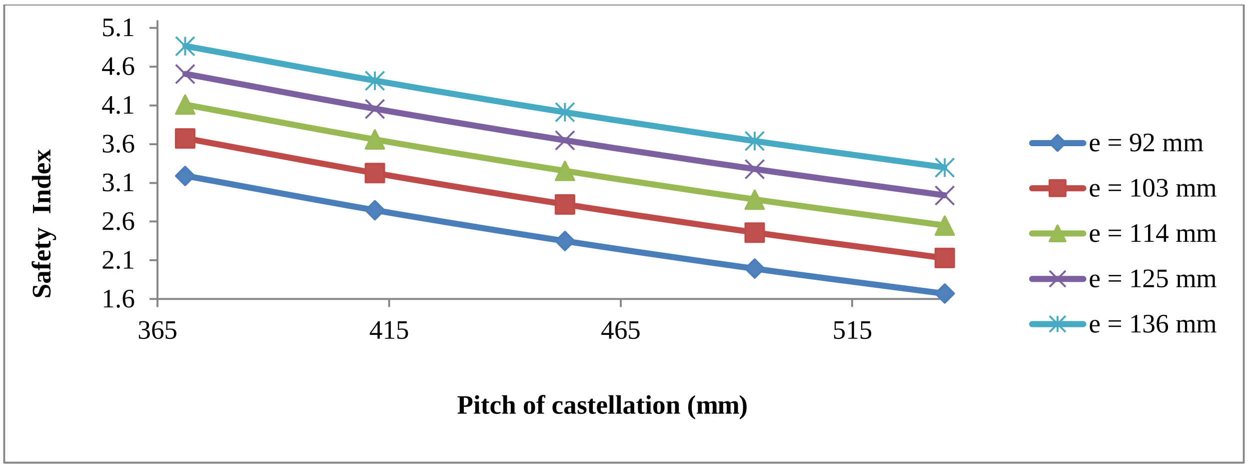

Fig. 11: Safety index against pitch of castellation for varying width of web post (Horizontal shear criterion)

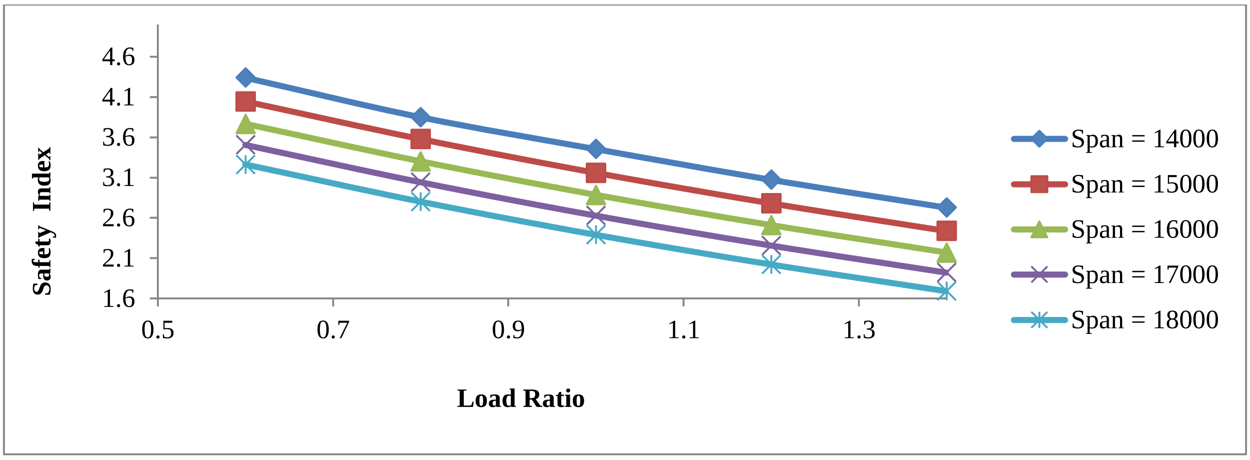

Fig. 12: Safety index against load ratio for varying beam span (Horizontal shear criterion)

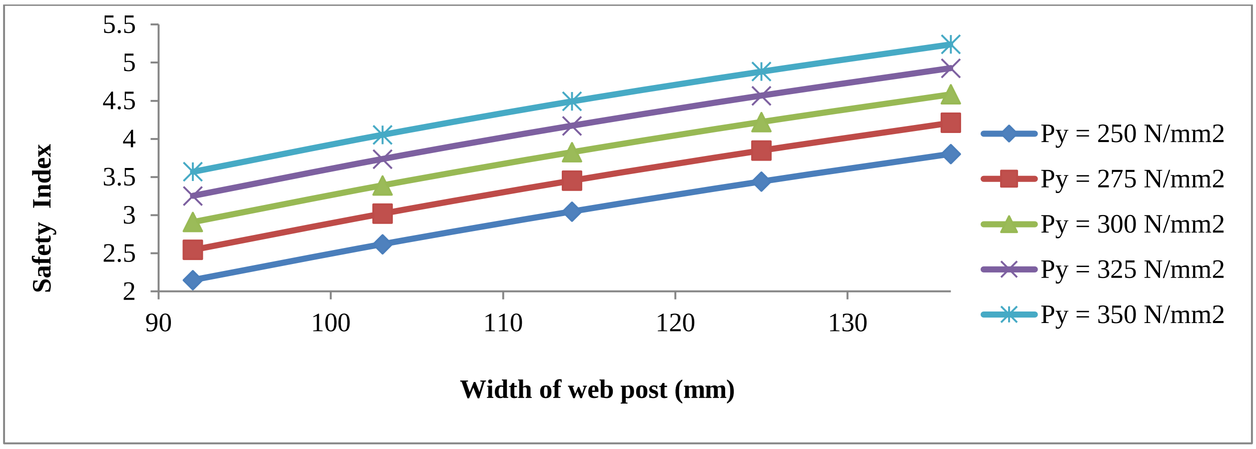

Fig. 13: Safety index against width of web post for varying bending strength (Horizontal shear criterion)

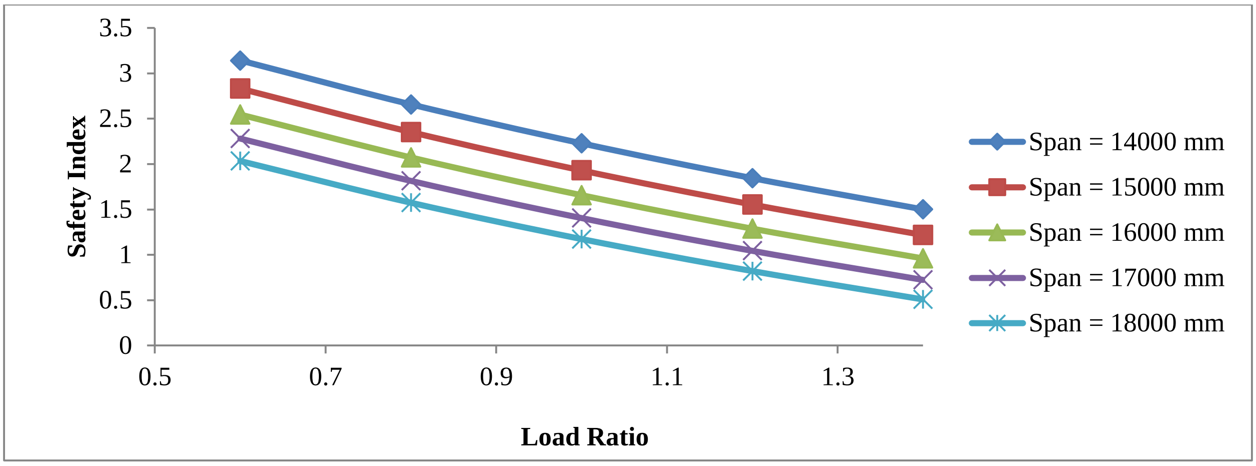

Fig. 14: Safety index against load ratio for varying beam span (Shear criterion)

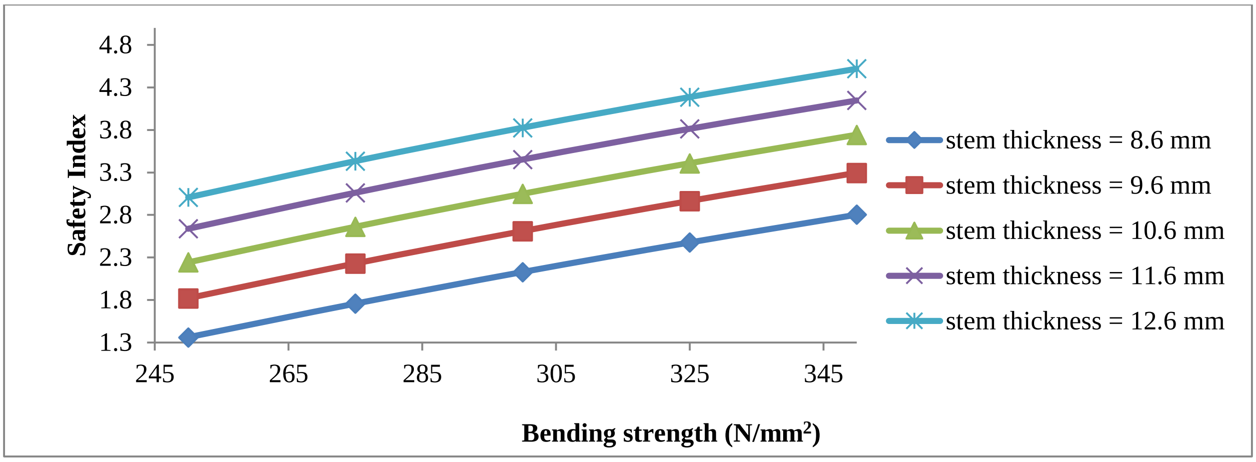

Fig. 15: Safety index against bending strength for varying thickness of stem of T-section (Shear criterion)

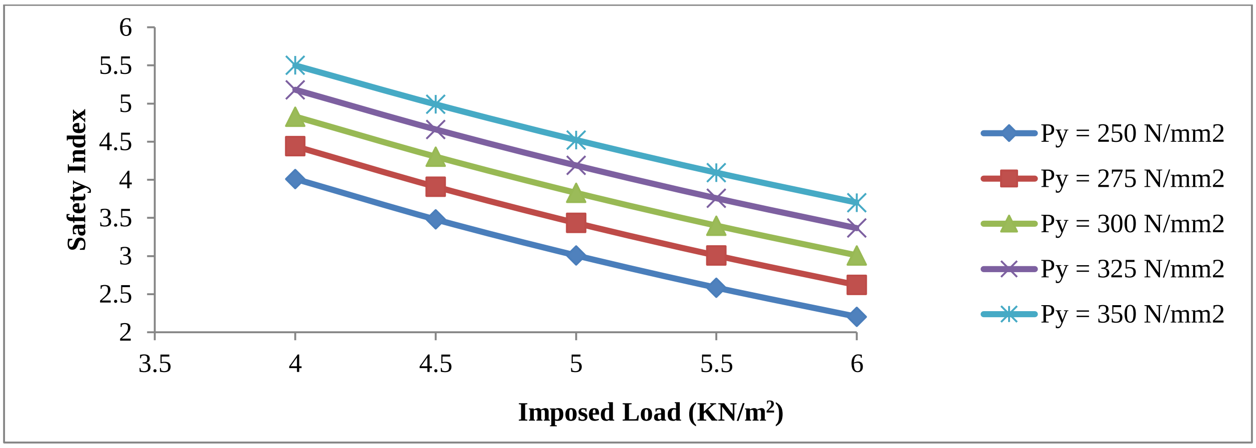

Fig. 16: Safety index against imposed load for varying bending strength (Shear criterion)

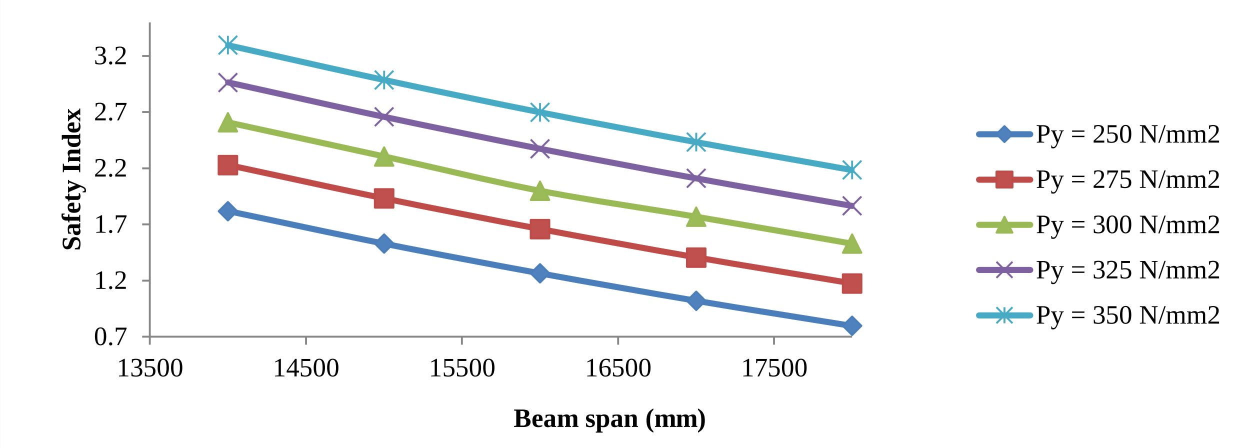

Fig. 17: Safety index against beam span for varying bending strength (Shear criterion)

From Figures 3 to 17, it can be seen that there is a general trend of safety level decreasing as the beam span and load ratio increase by 7.14% and 33.33% respectively, for all the failure criteria considered (Fig. 3, 5, 6, 9, 10, 12, 14 and 17). This trend is in agreement with Uche and Afolayan (2008) that the safety levels obtained based on BS 5950, Part 1(1990) is sensitive to load and resistance parameters. This trend could be attributed to increase in applied bending moment with increase in span of beam and load ratio. The safety level also increased by 4.19% as the area of T-sections increased by 2.87% with the implied safety indices ranging from 1.173 to 2.969 with an average value of 2.071 (Fig. 4). This could be attributed to increase in shear resistance of the beam with increase in area of T-sections. The safety level also increased by 42.8% as the distance between the centroids of the T-sections increased by 12.6% with the implied safety indices ranging from 0.817 to 3.498 with an average value of 2.158 (Fig. 7). This could be attributed to increase in depth of the beam with increase in distance between the centroids of the T-sections. The safety level decreased by 12.11% as the pitch of castellation increased by 11.05% with the implied safety indices ranging from 2.154 to 5.139 with an average value of 3.647 (Fig. 8 and 11). This could be attributed to the vulnerability of the web post to failure by buckling due to horizontal shear as the pitch of castellation increases (Tkalcevic et al., 2007). The safety level also increased by 21.9% as the width of web post increased by 11.95% with the implied safety indices ranging from 2.149 to 5.239 with an average value of 3.694 (Fig. 13). This is because increasing the width of the web post reduces the slenderness of the web thereby making it less vulnerable to web post buckling. The average values of the implied safety indices (3.647 and 3.694) from Figures 8, 11 and 13 for horizontal shear criterion fall within the range of the target safety index of 3.3 to 3.7 for structures of minor to large consequences of failure as recommended by the Joint Committee on Structural Safety (2001). The safety level also increased by 29.09% as the bending strength increased by 10% with the implied safety indices ranging from 1.361 to 4.521 with an average value of 2.941 (Fig. 15). This is because the moment capacity of the castellated beam increases as the bending or yield strength of the beam increases. The safety level also decreased by 13.26% as the imposed load increased by 12.5% with the implied safety indices ranging from 2.203 to 5.500 with an average value of 3.852 for varied values of bending strength of the beam (Fig. 16). This is because increase in imposed load value reduces the load bearing capacity of the castellated beam in bending. The beam span beyond 16m and a load ratio beyond 1.0 will not guarantee the safety of the beam as they gave negative values of safety indices (NKB, 1978; JCSS, 2001). The results of the reliability analysis have shown that the castellated beam is safe in horizontal shear, safe at some load ratios in shear but unsafe in bending.

5. Conclusions

The results of the safety analysis of a simply supported castellated beam with hexagonal openings with respect to the limit state of bending, shear and horizontal shear at web post in accordance with the design requirements of BS 5950 Part 1, 1985, have been presented. The safety indices generally decreased with increase in load ratio and beam span for all the failure modes considered. The safety indices increased with increasing area of T-sections and increased with increasing distance between the centroids of T-sections for bending criterion. The safety indices also decreased with increase in pitch of castellation and increased with increase in web post for horizontal shear criterion. The safety indices also increased with increase in bending strength and decreased with increase in imposed load for shear criterion. It was also shown that using a beam span beyond 16m and a load ratio beyond 1.0 will jeopardize the safety of the beam as they gave negative values of safety indices. The average values of the implied safety indices (3.647 and 3.694) for horizontal shear criterion fall within the range of the target safety index of 3.3 to 3.7 for structures of minor to large consequences of failure (NKB, 1978; JCSS, 2001). The results of the reliability analysis showed that the castellated beam is safe in horizontal shear, safe at some load ratios in shear but unsafe in bending.

References

Abhalesnehalsatish and Wakchaure, M.R. (2017) Study of castellated beams by using different codes. International Journal of Engineering Sciences and Research and Technology, 6(2): 748-751.

Abubakar, I. (2006) Reliability analysis of structural design parameters of reinforced concrete strip footings. Journal of Applied Science and Research, 2(7): 397-401.

Abubakar, I. and Peter, H.P. (2012) Failure investigation of Reinforced concrete cantilever retaining walls. Nigerian Journal of technology, 31(3): 248-260.

Abubakar, I. and Aliyu, I. (2017) Iso-safety design charts for singly reinforced concrete sections to Eurocode 0 recommended target safety indices. Jordan Journal of Civil Engineering 11(4): 525-533.

Afolayan, J.O. (2002) Parameter selection for safe design under uncertain conditions: A truss example. Journal of Engineering and Applied Science, 21: 77-82.

Amayrey, L. and Saka, M.P. (2005) Failure load prediction of castellated beams using artificial neural network. Asian Journal of Civil Engineering (Building and Housing), 6: 35-45.

Anupriya, B. and Jagadeesan, K. (2013) Strength study on castellated beam. International Journal of Engineering Research and Technology, 2(12): 3853-3859.

BS 5950 (1985) Structural use of steelwork in building, parts 1, British Standard Institution, London.

El-Reedy, M. (2013) Reinforced concrete structural reliability. Taylor and Francis, Boca Raton, FL.

JCSS (2001) Probabilistic model code. Publication of joint committee on structural safety, 12th draft.

Melchers, R. (1999) Structural Reliability Analysis and Prediction. Second Edition, John Wiley and Sons.

Mosley, W.H. and Bungey, J.H. (2002) Reinforced Concrete Design. 5th Edition, Macmillan Limited, London.

Nethercot, D.A. and Kerdal, D. (1982) Lateral-torsional buckling of castellated beams. Structural Engineering, 60B(3): 53-61.

NKB (1978) Recommendations for loading and safety regulations for structural design. Report, No. 36, Press Limited, London.

Ranganathan, R. (1999) Structural Reliability, Analysis and Design, Jaico Publishing House, Mumbai.

Redwood, R. and Demirdjian, S. (1998) Castellated beam web buckling in shear. ASCE Journal of Structural Engineering, 124: 1202-1207.

Shaikh, A.S. and Autade, P.B. (2016) Structural analysis and design of castellated beam in fixed actions. International Journal of Innovative Research in advanced Engineering, 8(3), 12: 2349-2763.

Sule, S. and Benu, M.J. (2019) Reliability analysis of a solid timber column subjected to axial and lateral loading, FUOYE Journal of Engineering and Technology, 4 (2): 1-5.

The Guardian Newspaper, 10th, December, 2016. Nigeria church collapse kills at least 160 worshippers.

Thoft-Christensen, P. and Baker, M.J. (1982) Structural reliability Theory and its Application, Berlin Springer-Verlag.

Tkalcevic, V., Dzeba, I. and Androic, B. (2007) Reliability of castellated beams subjected to lateral-torsional buckling, GRADEVINAR, 59(4): 311-318.

Uche, O.A.U. and Afolayan, J.O. (2008) Reliability-based rating for reinforced concrete column. Journal of Engineering and Technology, 3(1): 1-11.

Wakchaure, M.R. and Sagade, A.V. (2012) Finite element analysis of castellated steel beams. International Journal of Engineering and Innovative Technology, 2(1): 365-372.