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Probabilistic Analysis of a Rectangular Prestressed Concrete Beam

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Abstract
In this paper, the results of probabilistic analysis of a simply supported rectangular prestressed concrete beam under uniform loading at the limit state of bending, web-shear, flexure-shear and deflection are discussed. The design points and their corresponding reliability indices were obtained using a MATLAB program developed based on First Order Reliability format. The results of the sensitivity analysis carried out on the design variables showed that the reliability indices generally decreased with increase in beam span and load ratio for bending, web-shear, flexure-shear and deflection criterion and increased with increase in effective prestressing force for web-shear, flexure-shear and deflection criterion. The reliability indices were also found to increase with increase in load ratio and characteristic strength of tendons considering bending criterion and increased with increase in characteristic strength of concrete and width of beam considering flexure-shear criterion. The reliability indices were also found to be almost constant with increase in beam span and load ratio and almost constant with increase in eccentricity of prestressing force and load ratio considering deflection criterion. The results of the reliability analysis appeared to be safe in bending, unsafe in flexure-shear, unsatisfactory in web-shear and conservative in deflection when compared with the target reliability index value of 3.8 for safety class 2 at ultimate limit state.

Keywords: *Probabilistic analysis, Deflection, Web-shear, Flexure-shear, Design points, Sensitivity analysis, Prestressing force*

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1. Introduction

A prestressed concrete beam is one of the most widely used structural elements in the world. Prestressed concrete is one in which internal stresses of a suitable magnitude and distribution is introduced so that the stresses resulting from the external loads are neutralized to a desired degree (Krishna, 2006). Prestressed concrete is applied in buildings, bridges, offshore drilling platforms, etc (Lin and Burns, 1981). During prestressing, a compressive load is applied on a structural member to eliminate the internal tensile forces to eliminate cracks in the member implying that cracking is absent under working load. Consequently, lighter sections may be used to carry a given bending moment over a much longer span than the reinforced concrete counterpart (Reynolds and Steedman, 1997). In the design of a prestressed concrete beam subject to prestress, the stress distribution due to the prestress is combined with stresses due to the loading conditions in order to achieve permissible stress limits (Mosley and Bungey, 2002). The materials for the construction of prestressed concrete members are cement, concrete and steel.

The factors that affect the performance of prestressed concrete structures are uncertain. Abejide and Abejide (2014) investigated the safety of a prestressed concrete bridge beam in flexure. They found that the safety of post-tensioned concrete beam is sensitive to sectional modulus of concrete at the bottom, effective prestress force, profile eccentricity and beam span in flexure. The structural resistance and load effects are uncertain quantities. Due to their uncertain nature, the achievement of absolute structural safety remains an unattainable goal using a deterministic method of structural analysis. According to Afolayan (2002), the problems of civil engineering structures are stochastic in nature. Consequently, the use of partial safety factor in the design equations may lead to unsafe or conservative design of structures.

According to the publication of Joint Committee on Structural Safety (2002), structural failure is a probabilistic event. Structural reliability is therefore concerned with rational treatment of uncertainties. Probability and statistics always serve as a useful tool for dealing with uncertainties which are inherent in the design parameters rationally (Melchers, 1999; Sule, 2011; Abubakar, 2006; Ranganathan, 1999; El-Reedy, 2013; Thoft-Christensen and Baker, 1982). Many researchers (Abejide, 2014; Abubakar and Edache, 2007) have used probability and statistics to investigate the reliability of civil engineering structures.

In this paper, the probabilistic analysis of a rectangular prestressed concrete beam at the limit state of bending, flexure-shear, web-shear and deflection is carried out using First Order Reliability procedure. The failure functions were developed based on the failure modes, and were solved to obtain the design points on the failure surface and the corresponding reliability indices using a computer program written in MATLAB language.

2. Development of failure functions

The failure functions in bending, flexure-shear, web-shear and deflection were derived in accordance with the design requirements of BS 8110: part 1, 1985, for design of concrete structures. The beam (Figure 1) considered in this study is a simply supported prestressed concrete beam of rectangular cross section under uniform loading.

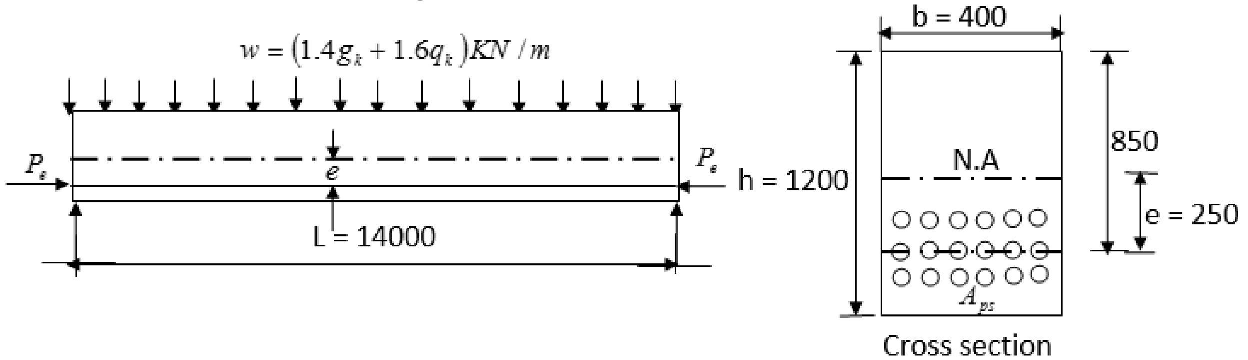

Fig. 1: A rectangular prestressed concrete beam under uniform loading

2.1 Bending criterion

The failure condition in bending is given by:

$$M_u \leq M_{app} \tag{1}$$

where M_u = ultimate moment of resistance of the beam, M_{app} = applied moment due to external load on the beam.

The maximum bending moment due to external load is given by:

$$M_{app} = \frac{wL^2}{8} \tag{2}$$

The applied moment at ultimate limit state is given by:

$$M_{app} = \frac{(1.4a + 1.6)q_k L^2}{8} \tag{3}$$

$$\alpha = \frac{g_k}{q_k} \tag{4}$$

The ultimate moment of resistance of a prestressed rectangular section is given by:

$$M_u = f_{pb} A_{ps} (d - 0.45x) \tag{5}$$

d = effective depth of the beam, x = depth to neutral axis.

The depth to neutral axis x is given by:

$$x = \lambda d \tag{6}$$

where λ = neutral axis depth parameter

Applying equations (3) and (5), the failure function in bending at ultimate limit state is given by:

$$G(x) = f_{pb} A_{ps} (d - 0.45\lambda d) - \frac{(1.4a + 1.6)q_k L^2}{8} \tag{7}$$

The stress in prestressed tendons at failure is given by:

$$f_{pb} = 0.95 f_{pu} \gamma \tag{8}$$

where γ = tendon parameter

Applying equation (8), equation (7) becomes:

$$G(x) = 0.95 f_{pu} \gamma A_{ps} (d - 0.45\lambda d) - \frac{(1.4a + 1.6)q_k L^2}{8} \tag{9}$$

According to BS 8110, part 1, 1985, $\lambda = 0.73$ and $\gamma = 0.67$

Equation (9) therefore becomes:

$$G(x) = 0.42741 f_{pu} A_{ps} d - \frac{(1.4a + 1.6) q_k L^2}{8} \quad (10)$$

2.2 Flexure-shear criterion

The failure condition in flexure-shear is given by:

$$M_o \leq M_{app} \quad (11)$$

where M_o = decompression moment

The decompression moment M_o is given by:

$$M_o = Z_b \left(\frac{P_e}{A} + \frac{P_e e}{Z_b} \right) \quad (12)$$

The applied moment at ultimate limit state is given by:

$$M_{app} = \frac{(1.4a + 1.6) q_k L^2}{8} \quad (13)$$

The failure function in flexure-shear is given by:

$$G(x) = Z_b \left(\frac{P_e}{A} + \frac{P_e e}{Z_b} \right) - \frac{(1.4a + 1.6) q_k L^2}{8} \quad (14)$$

where e = eccentricity of prestressing force, P_e = effective prestressing force

2.3 Web-shear criterion

The web-shear resistance of prestressed concrete section is given by:

$$V_{co} = 0.67 \sqrt{f_t^{\,2} + 0.80 f_{cp} f_t} \quad (15)$$

$$\text{where } f_{cp} = \frac{P_e}{bh} \quad (16)$$

$$\text{and } f_t = 0.24 \sqrt{f_{cu}} \quad (17)$$

The failure condition in web-shear is given by:

$$V_{co} \leq 0.5 V_{app} \quad (18)$$

The maximum applied shear stress in concrete at ultimate limit state is given by:

$$V_{app} = \frac{q_k (1.4a + 1.6) L}{2bd} \quad (19)$$

Applying Equations (15), (16), (17), (18) and (19), the limit state function in web-shear is given by:

$$G(x) = 0.335 \sqrt{0.0576 f_{cu} + 0.192 \frac{P_e}{bh} * \sqrt{f_{cu}}} - \frac{q_k (1.4a + 1.6) L}{4bd} \quad (20)$$

where f_t, f_{cp} = tensile strength of concrete and compressive prestress at the centroid of the section respectively, V_{co} = ultimate shear resistance of concrete in the section due to web-shear, f_{cu} = compressive strength of concrete.

2.4 Deflection criterion

The failure condition in deflection is given by:

$$\delta_{\max} \leq \delta_{all} \quad (21)$$

$$\text{where } \delta_{\max} = \frac{L}{500} \quad (22)$$

$$\delta_{all} = \frac{5L^2}{48Ey_t} \left(-\frac{M_{\max}}{Z_t} + \frac{P_e}{A} + \frac{P_e e}{Z_t} \right) \quad (23)$$

Applying Equation (21), the safety margin is given by:

$$G(x) = \frac{L}{500} - \frac{5L^2}{48Ey_t} \left(-\frac{M_{\max}}{Z_t} + \frac{P_e}{A} + \frac{P_e e}{Z_t} \right) \quad (24)$$

$$M_{\max} = M_{app} = \frac{(1.4a + 1.6) q_k L^2}{8} \quad (25)$$

Applying Equation (25), Equation (24) becomes:

$$G(x) = \frac{L}{500} - \frac{5L^2}{48Ey_t} \left(-\frac{(1.4a + 1.6) q_k L^2}{8Z_t} + \frac{P_e}{A} + \frac{P_e e}{Z_t} \right) \quad (26)$$

where y_t = distance from the centroidal axis to the extreme fiber in tension, δ_{all}, $\delta_{\max}$ = allowable deflection and maximum deflection due to applied load respectively.

3. Materials and methods

Let the failure surface in x-space be given by:

$$g(x) = g(x_1, x_2, \ldots, x_n) = 0 \quad (27)$$

The vector of the random variables in x-space corresponding to the limit state function is given by:

$$x = [x_1, x_2, \ldots, x_n]' \quad (28)$$

The normalized random variables $y_1, y_2, \ldots, y_n$ are introduced by a suitable one to one linear mapping in the form of $x = L(y)$ such that $y = L^{-1}(x)$. The corresponding design points in y-space is then defined by the transformation:

$$x = L(y), \quad y = L^{-1}(x) \quad (29)$$

Consequently, Equation (29) maps Equation (27) into:

$$h(y_1, y_2, \ldots, y_n) = 0 \quad (30)$$

The function h is defined by:

$$h(y) = g[L(y)] \quad (31)$$

Equation (31) is the failure function in normalized coordinate.

The minimum distance from the origin to the failure surface in normalized coordinate defines the reliability index, β. It is given by:

$$\beta = \min\left\langle \sqrt{\sum y_1^2 + y_2^2 + \ldots + y_n^2} \,\middle|\, h(y_1, y_2, \ldots, y_n) \right\rangle = 0 \quad (32)$$

In matrix notation, equation (32) can be re-written as:

$$\beta = \min\left\langle \sqrt{y^t y} \,\middle|\, h(y) \right\rangle = 0 \quad (33)$$

The values of the design variables that minimize the reliability index β subject to $h(y_1, y_2, \ldots, y_n) = 0$ are obtained by optimization. The flowchart of the optimization algorithm and the statistics of the basic variables are shown in Figure 2 and Table 1 respectively.

3.1 An example of a prestressed concrete rectangular beam

A 14 m simply supported prestressed Class 1 concrete beam of rectangular cross-section 400 mm x 1200 mm (Figure 1) was designed in accordance with the provisions of BS 8110 Part 1, 1985. The grade of concrete is C35; the Young's modulus of elasticity of concrete is 27kN/mm^2. 15.2mm 7-wire standard strand with tensile strength of 1700 N/mm^2 and y8 link were used as prestressed tendons and link reinforcement. The loss of prestress (Ls) is 25% and the imposed load on the beam is 15 kN/m. The statistics of the basic variables obtained from deterministic analysis are presented in Table 1.

Table 1: Statistics of basic variables

Basic Variable	**E(X)**	**Standard Deviation**	**COV(X)**	**PDF**
Area of prestressed steel, A_{ps}	3266 mm^2	489.9 mm^2	0.15	Lognormal
Effective depth of beam, d	850 mm	8.5 mm	0.01	Normal
Beam span, L	14000 mm	140 mm	0.01	Normal
Section modulus at top fiber, Z_t	96*10^6 mm^3	96*10^4 mm^3	0.01	Normal
Load Ratio, α	0.20	-	-	Fixed
Imposed load on beam, q_k	15 KN/m	4.5 KN/m	0.30	Gumbel
Eccentricity of prestressing force, e	250 mm	75 mm	0.30	Normal
Characteristic strength of prestressing steel, f_{pu}	1700 N/mm^2	255 N/mm^2	0.15	Lognormal
Area of cross-section of concrete, A	480,000 mm^2	72,000 mm^2	0.15	Normal
Effective prestressing force, P_e	2772000 N	831600 N	0.30	Gumbel
Modulus of elasticity of concrete, E	27*10^5 N/mm2	27*10^3N/mm^2	0.01	Normal
Moment of inertia of concrete, I	6.912*10^13 mm^4	6.912*10^11 mm4	0.01	Normal
Distance from the centroidal axis to the extreme fiber in tension, y_t	600 mm	6 mm	0.01	Normal
Width of prestressed beam, b	400 mm	4 mm	0.01	Normal
Characteristic strength of concrete, f_{cu}	35 N/mm^2	5.25 N/mm^2	0.15	Lognormal

Source: (Abejide and Abejide, 2017; Ranganathan, 1990; Abubakar and Aliyu, 2017; Abubakar et al., 2014; Cavaco et al., 2010; Hamidu, 2006)

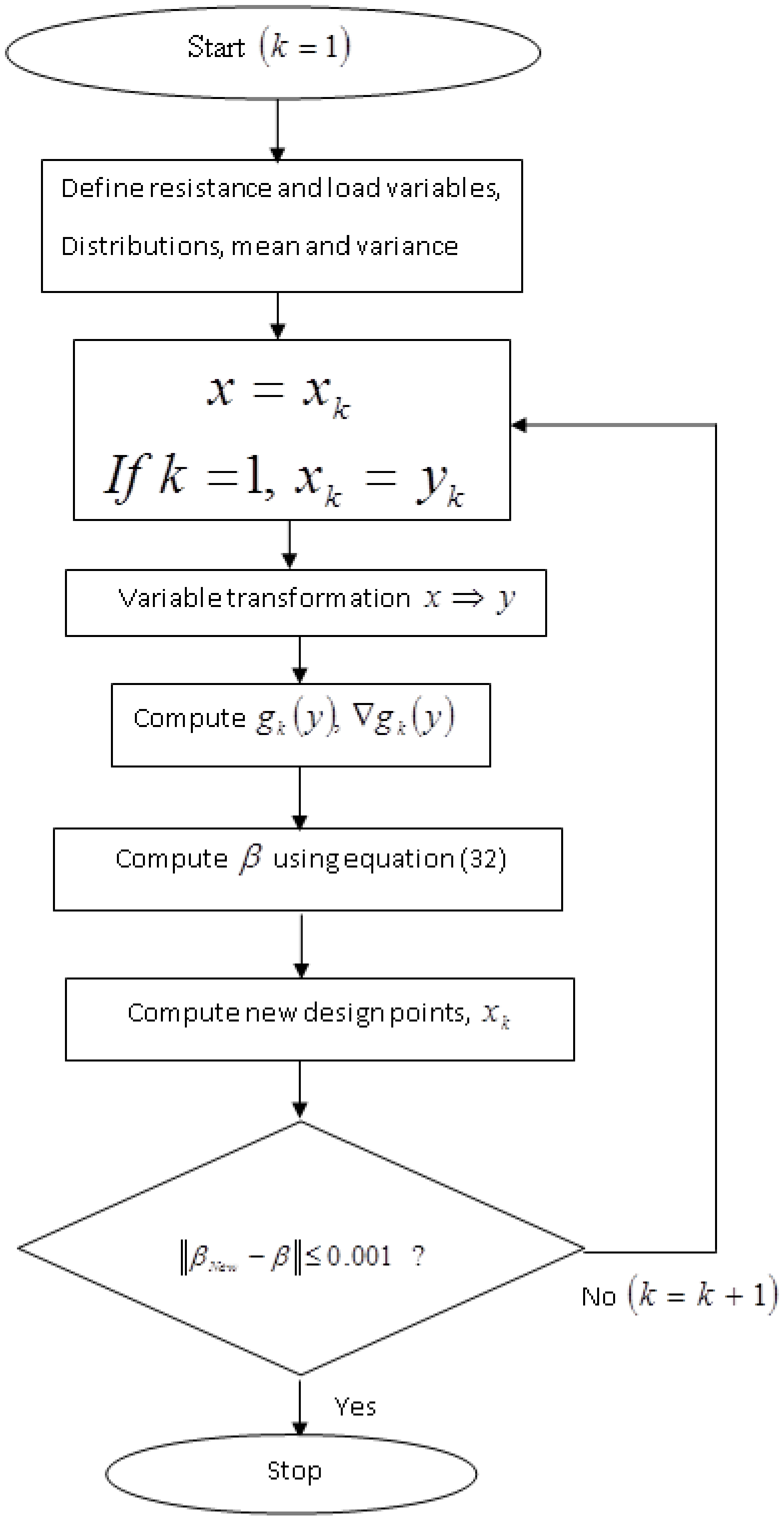

Fig. 2: Flowchart showing the optimization algorithm

4. Results and discussion

The design points on the failure surface and the reliability indices for the various failure modes considered were obtained using a MATLAB program. Figures 3 to 6 show the relationship between the reliability indices and varied values of the random variables for bending failure mode; Figures 7 to 9 show the relationship between the reliability indices and varied values of random variables for flexure-shear failure modes; Figures 10 to 12 show the relationship between the reliability indices and varied values of the random variables for web-shear failure mode and Figures 13 to 16 show the relationship between the reliability indices and varied values of the random variables for deflection failure mode. It can be observed from Figures 3 to 16 that:

a. The reliability indices generally decrease with increase in load ratio and beam span for all the failure modes considered. This is because the applied bending moment increases with increase in load ratio and beam span leading to the reduction of flexural strength of the beam.
b. Increasing the beam span beyond 16m will jeopardize the safety of the beam in flexure-shear. The negative value of the reliability index in flexure-shear shows that 17m span is not safe (EN 1990, 2002).
c. Increasing the load ratio and characteristic strength of tendons increases the reliability of the beam considering bending criterion.
d. Increasing the effective prestressing force and eccentricity of prestressing force increases the reliability of the beam considering flexure-shear criterion. This is in agreement with the results obtained by Abejide and Abejide (2014).
e. Increasing the effective depth of the beam and effective prestressing force increase the reliability of the beam for web-shear failure mode. This is because as the effective depth of the beam increases, the overall depth of the beam increases and the shear stress on the beam reduces.
f. Increasing the characteristic strength of concrete and width of beam for web-shear criterion increases the reliability of the beam.
g. Increasing the beam span and effective prestressing force increase the reliability of the beam for deflection criterion. This is because prestressed concrete beam of longer span has good deflection capacity irrespective of the magnitude of the beam loading.
h. The reliability indices are almost constant with increase in beam span and eccentricity of prestressing force for deflection criterion. This behavior may be due to the fact that the load ratio has little effect on the failure of the beam due to deflection.
i. The results of the reliability analysis appeared to be safe in bending, unsafe in flexure-shear, unsatisfactory in web-shear and conservative in deflection when compared with the target reliability index value of 3.8 for Safety class 2 (EN1990, 2002).

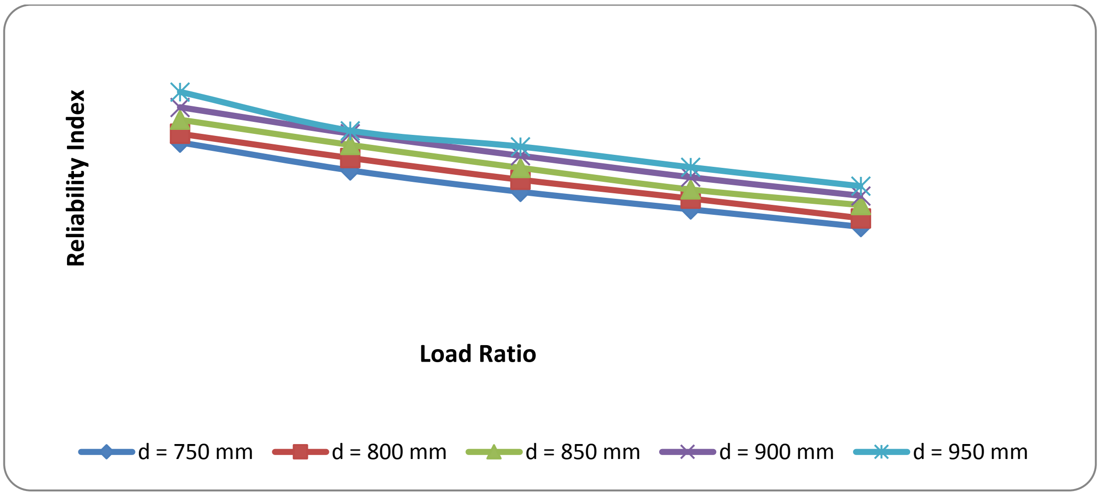

Fig 3: Reliability index against load ratio for varying effective depth of tension bars (Bending criterion)

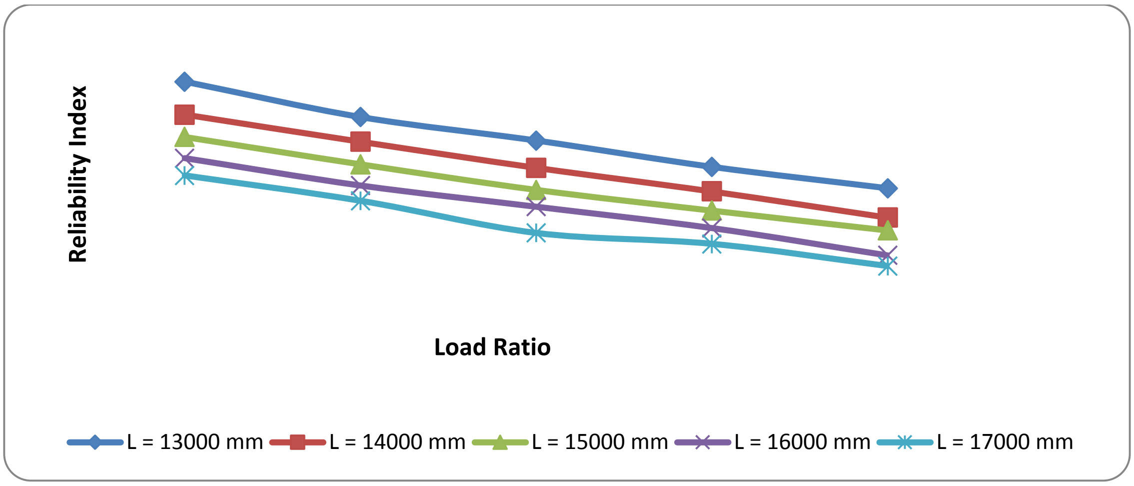

Fig. 4: Reliability index against load ratio for varying beam span (Bending criterion)

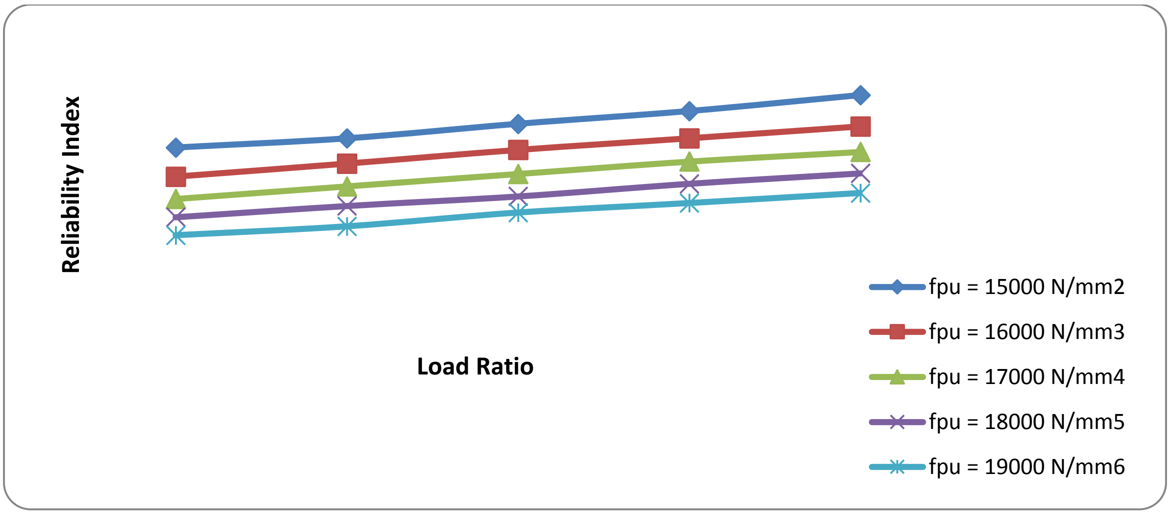

Fig. 5: Reliability index against load ratio for varying characteristic strength of prestressing steel (Bending criterion)

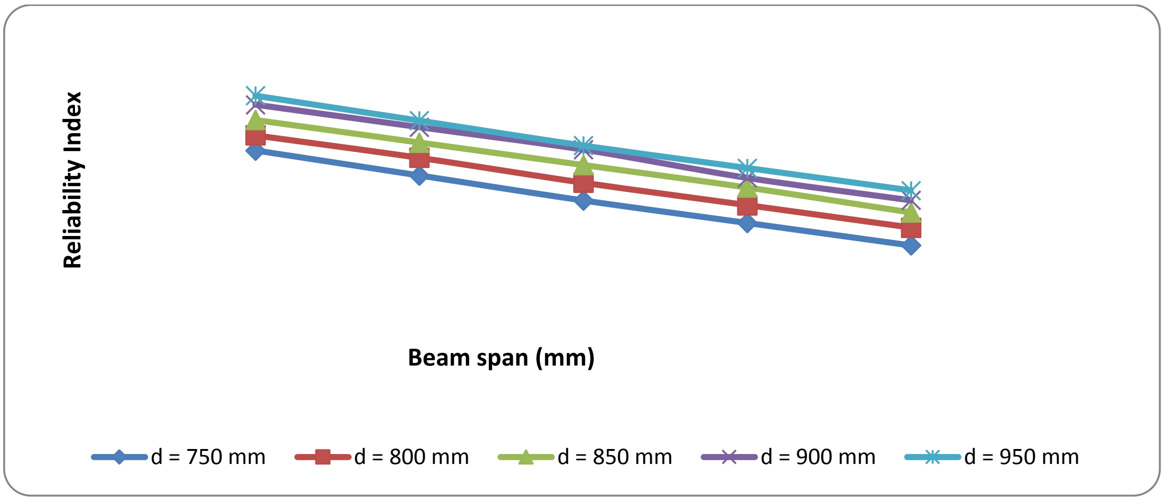

Fig. 6: Reliability index against beam span for varying effective depth of tension bars (Bending criterion)

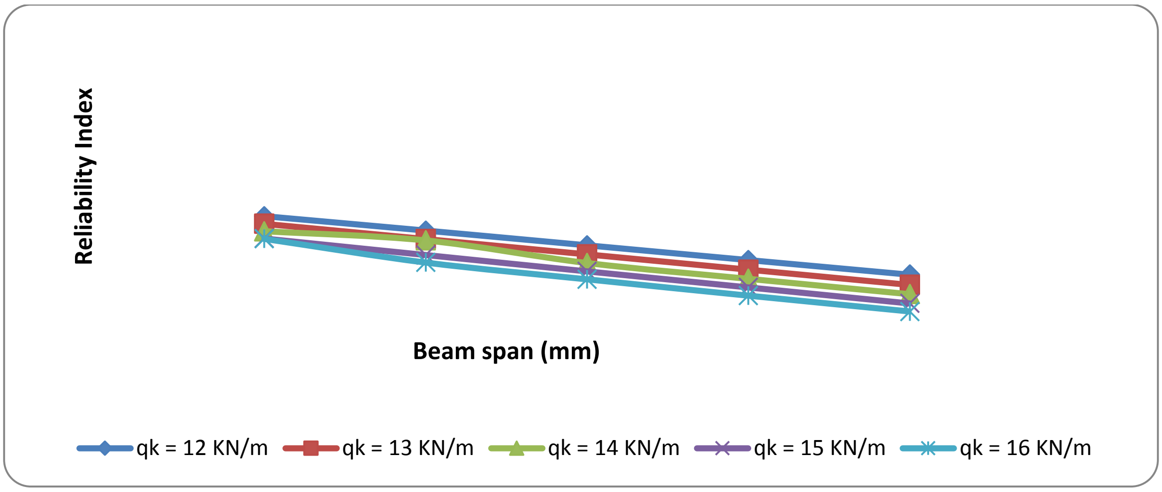

Fig. 7: Reliability index against beam span for varying imposed loads (Flexure-shear criterion)

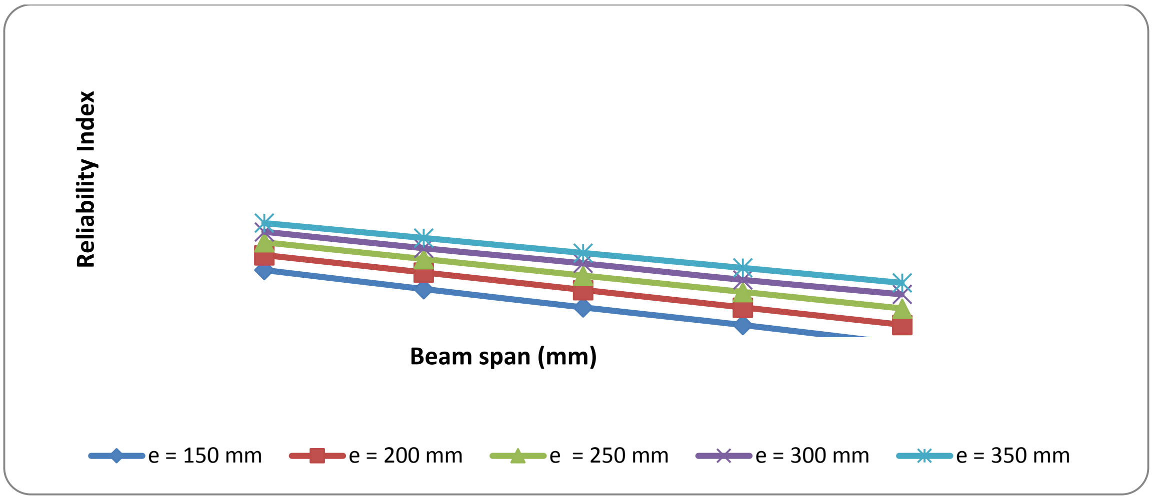

Fig 8: Reliability index against beam span for varying eccentricity of prestressing force (Flexure-shear criterion)

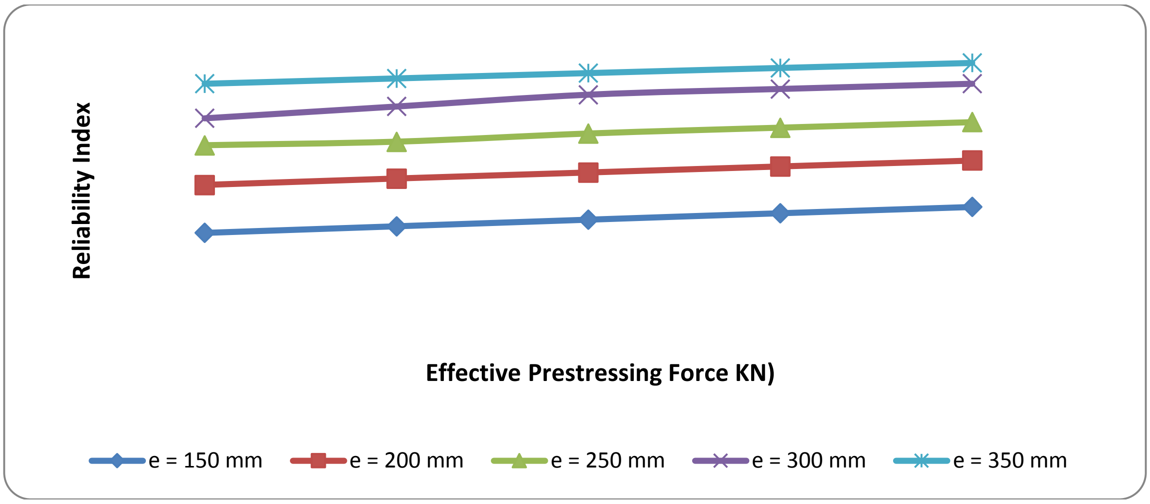

Fig. 9: Reliability index against effective prestressing force for varying eccentricity of prestressing force (Flexure-shear criterion)

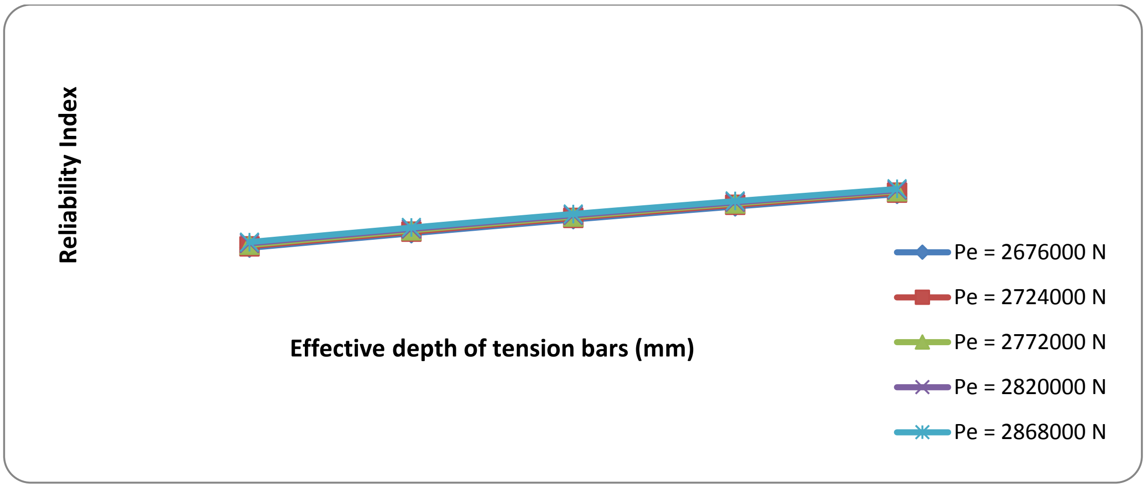

Fig. 10: Reliability index against effective depth for varying effective prestressing force (Web-shear criterion)

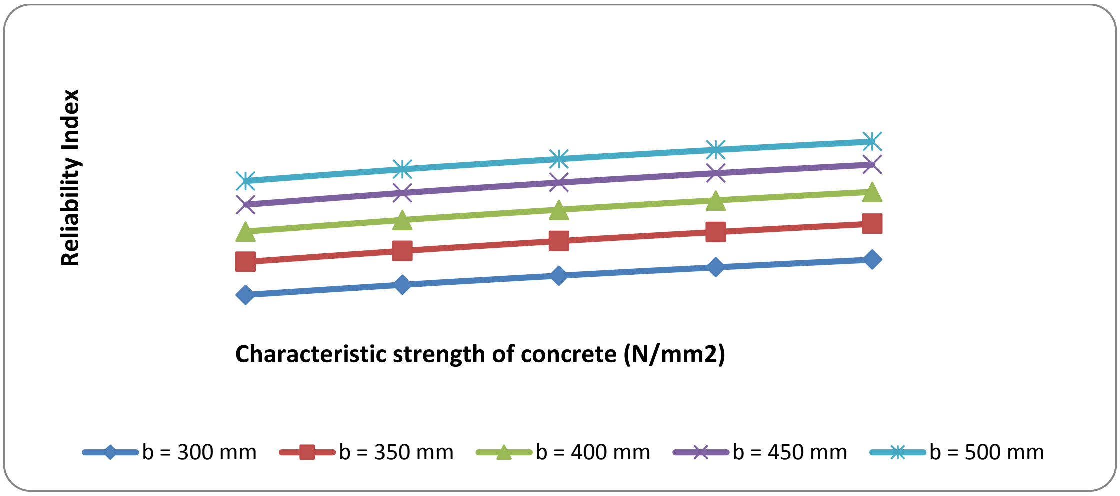

Fig. 11: Reliability index against characteristic strength of concrete for varying width of beam (Web-shear criterion)

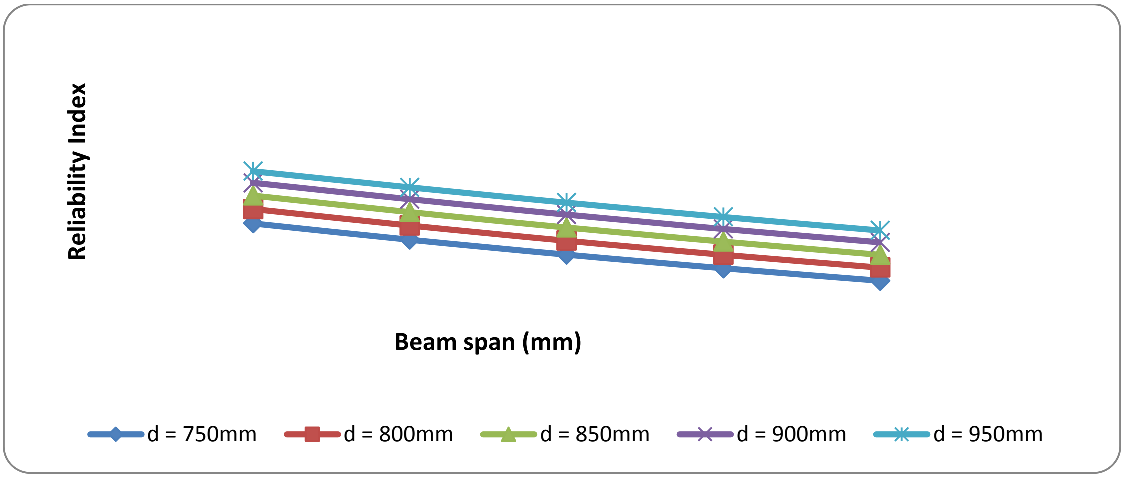

Fig. 12: Reliability index against beam span for varying effective depth of tension bars (Web-shear criterion)

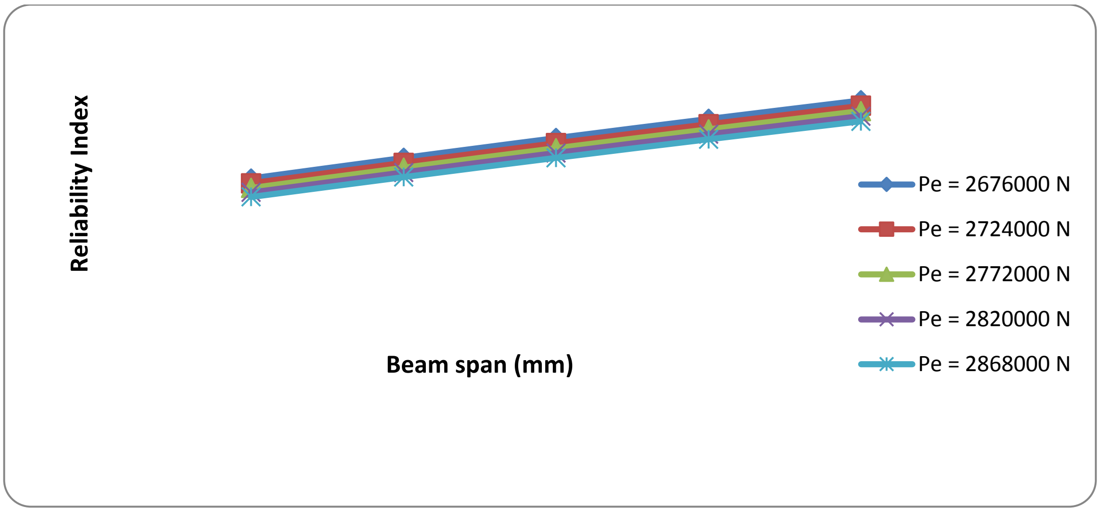

Fig. 13: Reliability index against beam span for varying effective prestressing force (Deflection criterion)

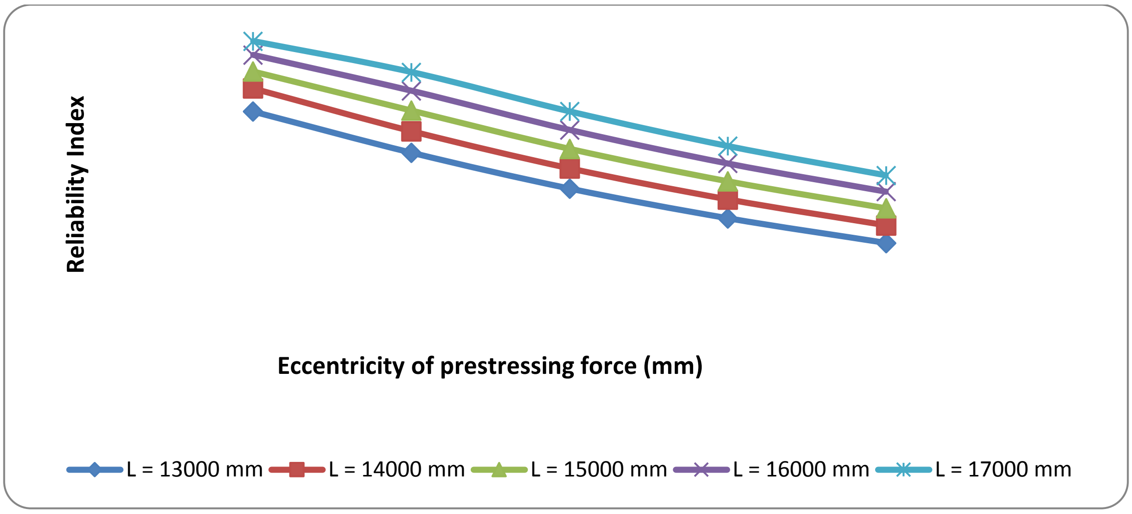

Fig. 14: Reliability index against eccentricity of prestressing force for varying beam span (Deflection criterion)

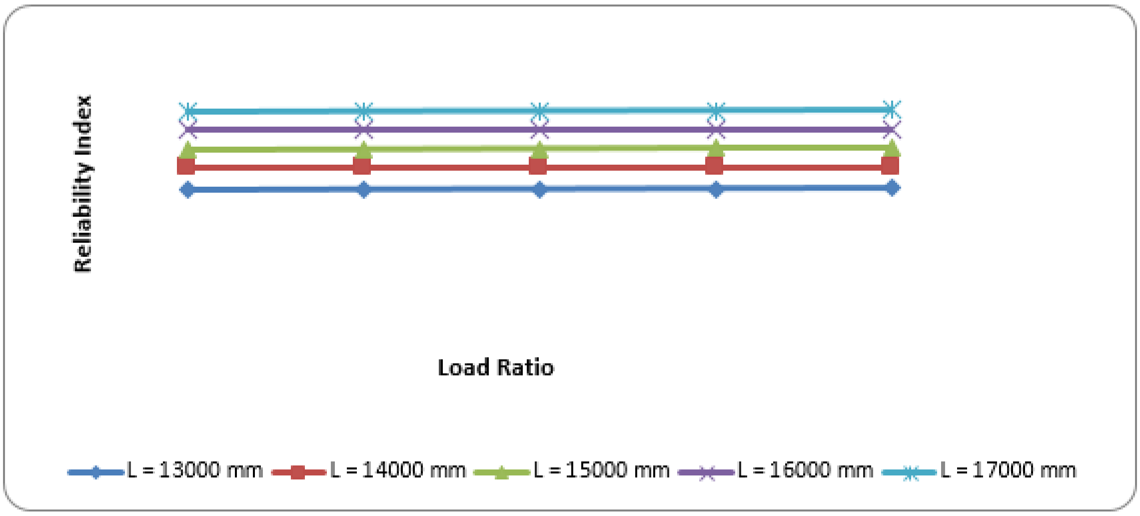

Fig. 15: Reliability index against load ratio for varying beam span (Deflection criterion)

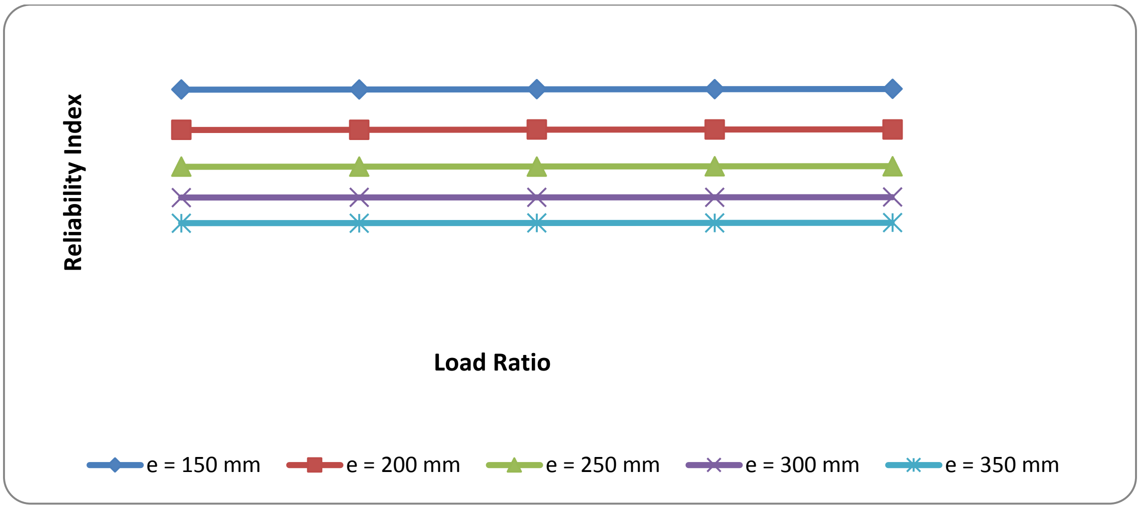

Fig. 16: Reliability index against load ratio for varying eccentricity of prestressing force (Deflection criterion)

5. Conclusions

The results of the safety analysis of a simply supported prestressed concrete beam with bonded tendons under uniform loading at the limit state of bending, web-shear, decompression moment and deflection have been presented. The results of the sensitivity analysis carried out on the design variables showed that the reliability indices generally decreased with increase in beam span and load ratio for bending, web-shear, flexure-shear and deflection criterion and increased with increase in effective prestressing force considering web-shear, flexure-shear and deflection criterion. The reliability indices also increased with increase in load ratio and characteristic strength of tendons for bending criterion and increased with increase in characteristic strength of concrete and width of beam for flexure-shear criterion. The reliability indices were also found to be almost constant with increase in beam span and load ratio and almost constant with increase in eccentricity of prestressing force and load ratio considering deflection failure mode. The design reliability levels seemed to be safe in bending, unsafe in flexure-shear, unsatisfactory in web-shear and conservative in deflection when compared with the target reliability index value of 3.8 for Safety class 2 at ultimate limit state.

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