

Crude Oil Value Chain Optimization Scenarios: Lessons from Nigeria

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Abstract

This paper optimizes the allocation of Nigeria's projected oil production and refined products under three different possible scenarios - Business as Usual, Stated Policy, and Energy Transition. These scenarios capture the uncertainties offered by future oil production, increasing domestic products demand, energy prices, timing of domestic refining capacity buildup, and the calls for global transition from fossils. Based on the Reference Energy System developed by Gbakon et. al. (2021) for crude oil flow through the integrated oil value chain, the net benefit objective function is developed. The integrated oil value chain is optimized by maximizing the net benefit function under the different scenarios. Extending the framework by a Monte Carlo formulation of the problem allows greater flexibility in addressing questions of the likelihood of attaining policy outcomes such as product self-sufficiency. A family of curves is generated within the solution structure, representing the confidence interval within which policy performance outcomes can be located. Scenario analysis for example shows that under the "Energy Transition" scenario, net benefit of \$ 192 billion is realized. Whereas, under the "Business-as-Usual" scenario the net benefit is \$ 423 billion. Under the "Stated Policy" scenario, the net benefit is \$ 718 billion. Implications for net system benefits and the respective drivers are further interrogated. The need to optimally allocate Nigeria's future oil production and resulting refined products to diverse end-use cannot be over-emphasized. Midstream infrastructure such as refineries, pipelines and storage are critical to achieve optimal performance in the value chain. This will have impact on expected oil export earnings, domestic fuels' imports, and the potential for petroleum products' export earnings.

Keywords: Energy system modelling, Crude oil value chain optimization, Energy transition; Monte Carlo simulation, Nigeria

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1. Introduction

The crude oil value chain comprises the physical and commercial systems through which crude oil flows from well to wheels. It typically includes upstream facilities for the exploration, development and production of oil, pipelines, sea going tankers, storage facilities, refineries, terminals, and the associated commercial agreements. Between 80% and 90% of crude oil refined ends up as transport fuels (EIA, 2022). These diverse range of activities in the value chain are sequenced operationally and economically to convert crude oil into transport fuels, electric power, or petrochemicals for end user consumption. The petroleum value chain is segregated into Upstream, Midstream and Downstream. The Upstream involves the exploration, development, and production of oil and

gas; The Midstream involves oil and/or gas transportation, storage, and processing – refining, gas processing; while the Downstream involves retail, power, and gas utilization.

Between 1979 and 1989, Nigeria's oil exports as a percentage of oil production, rose rapidly from 76% to 89% and reached ~ 100% in 2009. Meanwhile refinery capacity utilization declined from 70% (2000) to 0% (2018) with very volatile swings in between. Demand for petroleum products in Nigeria, consisting mostly of the transport fuels of gasoline, diesel, and kerosene, increased from estimated 169 Mbpd (~27 MM litres/day) in 1995 to 440 Mbpd (70 MM litres/day) in 2018. Due to the low refinery capacity utilization coupled with increasing fuel demand, 96% of the petroleum product demand was met by imports and/or OPA

(Offshore Processing Arrangements) or RPEA (Refined Product Exchange Arrangements) between 2010 to 2020 (Sayne et al., 2015). Between 2010 and 2020, Nigeria's oil production declined by 31% from 930 MMbbls in 2010 to 644 MMbbls in 2020. Yet, export of a higher proportion of declining production, continued with unstable and declining domestic refining capacity utilization in the background (Fig. 1).

The quartet of declining upstream production, the increased ratio of oil exported to production, low domestic refining capacity utilization, and increasing petroleum demand gives rise to the question of whether Nigeria's crude oil value chain is optimized. The fundamental problem addressed in this paper is how oil production and subsequently refined products can be optimally allocated for the different end uses in such a manner as to maximize producer and consumer surplus.

2. Materials and methods

According to Hoffman & Wood (1975), the development of energy system model is an endeavour which calls on the theoretical and analytical methods from several disciplines. Multiple disciplines including engineering, economics, operations research, and management science are all marshalled behind building and interpreting energy system models. According to the fifth Assessment Report of the IPCC (Intergovernmental Panel on Climate Change), an energy system "comprises all components related to the production, conversion, delivery, and use of energy." Of the several techniques employed for energy system modelling, Sydsæter, Hammond, & StrOm (2012) single out the optimization method as the most popular due to the ease to which it lends itself to provide economically meaningful interpretation. The technique finds use in different domains – specific to oil and gas, it has been applied to upstream production operations (Kaufman et al., 2020; Ghaelia, 2019; Gao, 2009; Aziz, 2002; Wang et al., 2002;), refinery production (Murty, 2020; Ejikeme-Ugwu, 2012, Chairat, 1971), and oil and gas portfolio optimization (Huang, 2019; Domnikov et al., 2017; Aibassov, 2007). Other areas of application have been in financial asset portfolio optimization (Qing

et. al., 2014; April, et. al., 2003; Oladejo et. al., 2020), factory production scheduling (Kimutai et. al., 2019)

The use of optimization is well represented with a long and rich background. For the construction of sectoral to (sub)-national energy system models, Al – Amer et. al. (1998) formulated a mixed integer linear program for the development of petrochemical industry in Saudi Arabia, which objective was the maximization of profit. Ye et. al. (2019) optimized a regional integrated energy supply system consisting of coal, gas and heat by minimizing the total cost of construction and operation under the energy demand constraints and energy network security requirements. Beiranvand et. al. (2018) formulated a mathematical model of Iran's petroleum supply chain supply under uncertain demand and price. Beiranvand et. al. (2018), under five scenarios of demands and prices, minimized system cost subject to constraints such as material balance, oil production, refinery throughputs, and storage capacities amongst others. Adegbulugbe, Dayo and Gurtler (1989) estimated the long-term optimal structure of the Nigerian energy supply mix over a 30-year horizon (1980 – 2010) in 5-year steps by minimizing total direct fuel costs (operating and maintenance, transportation or transmission, and investment) as the objective function. Salehi and Goorkani (2017) formulated a stochastic, linear multi-objective model to optimally allocate Iranian oil and gas resources under condition of imposed sanctions.

2.1 Methodology

For this paper, the objective is to optimize the allocation of oil production and refined products to meet increasing transport fuels demand, subject to capacity constraints. This work relies on the network depiction in Fig. 1 showing the flow of oil resources either via imports or domestic production, through conversion to various petroleum products to meet domestic demand. Both the deterministic-based scenario and probabilistic analysis conducted are based on the Reference Energy System (Beller, 1976), developed specifically for the allocation of oil production (a sub-sector energy system). The optimization model of this paper is derived from the network schematic in Fig. 1.

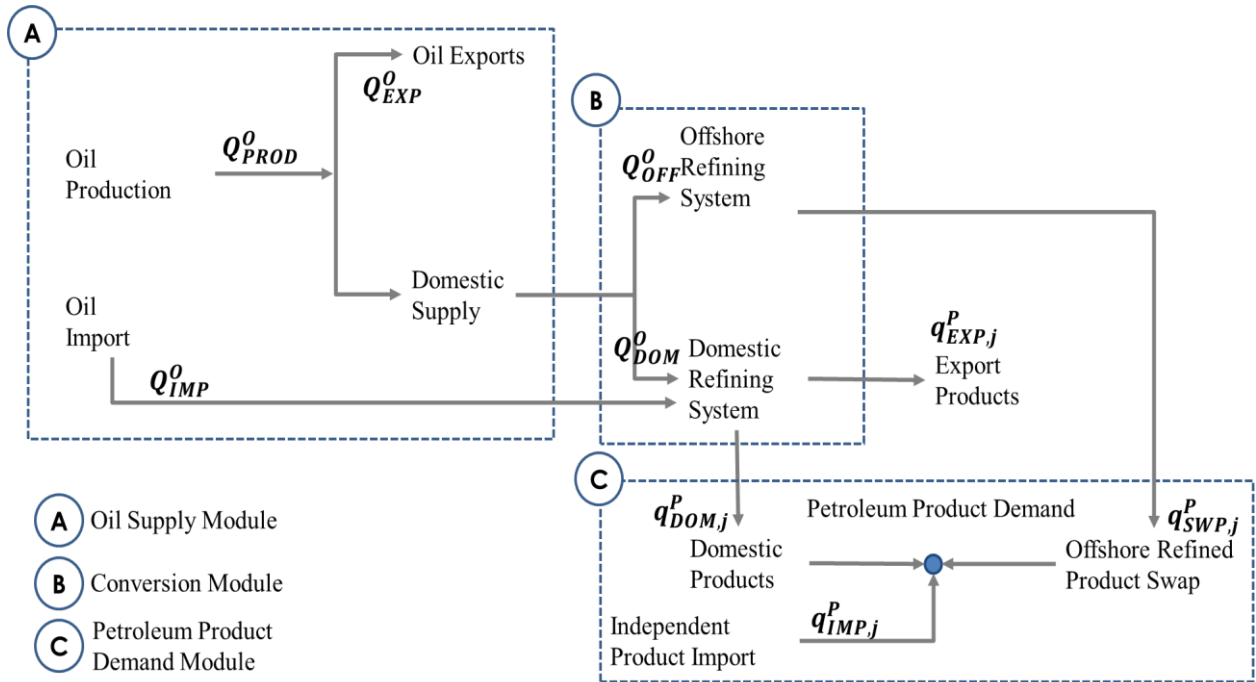


Fig. 1: Reference energy system for crude oil utilization

2.2 Optimization model

The optimization model developed to address the central objective of this paper is given by the general framework:

$$\begin{aligned} & \text{Maximize } Z = C^T X \\ & \text{Subject to } AX \leq b \\ & X \geq 0 \end{aligned} \quad (1)$$

where, $Z = C^T X$ is the Objective Function, $AX \leq b$ represents the functional constraint, and $X \geq 0$ is the non-negative constraint. The symbols used in the model are explained as follows: P^O is the Price of Crude Oil (\$/bbl), ΔP^O is the quality differential for crude oil imported (\$/bbl), $P_{EXP,j}^P$ is the Price of Refined Product j for Export (\$/bbl), $P_{DOM,j}^P$ is the Price of Product j to domestic (\$/bbl), $P_{IMP,j}^P$ is the Price of Product j in the source market (to be imported) (\$/bbl), C_{DIST}^P is the cost of product distribution to domestic (\$/bbl), $C_{LOSS,j}^P$ is the cost of loss of j th product distribution to domestic (\$/bbl), C_{DIST}^O is the cost of oil distribution to domestic (\$/bbl), $C_{EMIT,j}^{CO2}$ is the cost of carbon emissions from domestic end use of fuel j (\$/bbl), C_{PROD}^O is the cost of upstream oil production (\$/bbl), C_{PROD}^{CO2} is the cost of carbon emissions from upstream production (\$/bbl), C_{DT}^O is the Dirty Tanker freight (oil shipping) (\$/bbl), C_{CT}^P is the Clean Tanker freight (product shipping) (\$/bbl), C_{DREF}^O is the variable cost of domestic refining

(\$/bbl), C_{DREF}^{CO2} is the cost of carbon emissions from domestic refining (\$/bbl), C_{OREF}^O is the processing fee for offshore refining (\$/bbl), C_{LOSS}^O is the cost of crude oil loss (\$/bbl), FC_{DREF} is the Fixed Cost of domestic refining (\$MM), FC_{DIST} is the Fixed Cost of domestic distribution (\$MM), Q_{PROD}^O is the upstream crude oil production (MMbbls), Q_{EXP}^O is the crude oil exported (MMbbls), Q_{DOM}^O is the crude oil for domestic refining (MMbbls), Q_{OFF}^O is the crude oil to offshore refining (MMbbls), Q_{IMP}^O is the crude oil imported into the domestic refining system (MMbbls), $q_{DOM,j}^P$ is the j th product from domestic refining into the domestic market (MMbbls), $q_{EXP,j}^P$ is the j th product from domestic refining which is exported (MMbbls), $q_{SWP,j}^P$ is the j th Product from offshore refining/swap (MMbbls), $q_{IMP,j}^P$ is j th product imported independently into the domestic market (MMbbls), $q_{DEM,j}^P$ is the j th product demand of the domestic market (MMbbls), $TDRC$ is the Total Domestic Refining Capacity (MMbbls), $TORC$ is the Total Offshore Refining Capacity (MMbbls), and $j = 1 \dots 5$ is the subscript representing the five (5) different petroleum products – LPG, Gasoline, Diesel, Kerosene, and Fuel Oil. The objective is to maximize the profit (or net benefit) of the system, which is the difference between the "Inflows" and "Outflows" summed up across the nodes of the network.

The Objective Function

The Inflow is given by Equation (2) as:

$$\sum_{t=1}^T \text{INFLOW}_t = \sum_{t=1}^T \left[P_t^O [Q_{\text{EXP},t}^O + Q_{\text{DOM},t}^O] + \sum_{j=1}^5 P_{\text{EXP},j,t}^P [q_{\text{EXP},j,t}^P] + \sum_{j=1}^5 P_{\text{DOM},j,t}^P [q_{\text{DOM},j,t}^P + q_{\text{SWP},j,t}^P + q_{\text{IMP},j,t}^P] \right] \quad (2)$$

There are three components to Equation 2, The first, $\sum_{t=1}^T P_t^O [Q_{\text{EXP},t}^O + Q_{\text{DOM},t}^O]$, represents the receipts from the sale of crude oil to the export market and to domestic refining. It has been assumed that the same price is received for exports as for domestic sales. The second, $\sum_{t=1}^T \sum_{j=1}^5 P_{\text{EXP},j,t}^P [q_{\text{EXP},j,t}^P]$, accounts for the inflows from exporting domestically refined products. The third

component, $\sum_{t=1}^T \sum_{j=1}^5 P_{\text{DOM},j,t}^P [q_{\text{DOM},j,t}^P + q_{\text{SWP},j,t}^P + q_{\text{IMP},j,t}^P]$, aggregates the proceeds from the sale of refined products into the domestic market by three means. The formula recognizes the three sources of refined products into the domestic market. Equation (3) represents the Outflow of the system.

$$\begin{aligned} \sum_{t=1}^T \text{OUTFLOW}_t = \sum_{t=1}^T \left[& Q_{\text{EXP},t}^O [C_{\text{PROD},t}^O + C_{\text{PROD},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O] \right. \\ & + Q_{\text{OFF},t}^O [C_{\text{DT},t}^O + C_{\text{OREF},t}^O + C_{\text{PROD},t}^O + C_{\text{PROD},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O] + Q_{\text{DOM},t}^O [C_{\text{DIST},t}^O \\ & + C_{\text{DREF},t}^O + C_{\text{PROD},t}^O + C_{\text{PROD},t}^{\text{CO}_2} + C_{\text{DREF},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O] \\ & + Q_{\text{IMP},t}^O [P_t^O + \Delta P_t^O + C_{\text{DIST},t}^O + C_{\text{DREF},t}^O + C_{\text{DREF},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O] \\ & + \sum_{j=1}^5 q_{\text{SWP},j,t}^P [C_{\text{CT},j,t}^P + C_{\text{DIST},j,t}^P + C_{\text{EMIT},j,t}^{\text{CO}_2} + C_{\text{LOSS},j,t}^P] \\ & + \sum_{j=1}^5 q_{\text{IMP},j,t}^P [P_{\text{IMP},j,t}^P + C_{\text{CT},j,t}^P + C_{\text{DIST},j,t}^P + C_{\text{EMIT},j,t}^{\text{CO}_2} + C_{\text{LOSS},j,t}^P] \\ & + \sum_{j=1}^5 q_{\text{DOM},j,t}^P [C_{\text{DIST},j,t}^P + C_{\text{EMIT},j,t}^{\text{CO}_2} + C_{\text{LOSS},j,t}^P] + \sum_{j=1}^5 q_{\text{EXP},j,t}^P [C_{\text{LOSS},j,t}^P] \\ & \left. + \text{FC}_{\text{DREF},t} + \text{FC}_{\text{DIST},t} \right] \quad (3) \end{aligned}$$

The "Outflow" equation has nine components. The first, $\sum_{t=1}^T Q_{\text{EXP},t}^O [C_{\text{PROD},t}^O + C_{\text{PROD},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O]$, captures the associated cost of producing oil that goes towards export. The cost of upstream production, carbon costs associated with upstream production and the cost of losses. The second, $\sum_{t=1}^T Q_{\text{OFF},t}^O [C_{\text{DT},t}^O + C_{\text{OREF},t}^O + C_{\text{PROD},t}^O + C_{\text{PROD},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O]$, represents the cost of sending oil to be refined in an offshore refinery. Cost of upstream production, cost of carbon associated with upstream production, freight costs to the offshore refinery, offshore refinery fees and losses. The third, $\sum_{t=1}^T Q_{\text{DOM},t}^O [C_{\text{DIST},t}^O + C_{\text{DREF},t}^O + C_{\text{PROD},t}^O + C_{\text{PROD},t}^{\text{CO}_2} + C_{\text{DREF},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O]$, reflects the cost associated with the delivery of oil to the domestic refining system. Cost of upstream production, cost of carbon associated with upstream operations, cost of transport to the refinery, cost of domestic

refining, carbon costs associated with the refining system and losses. The fourth term, $\sum_{t=1}^T Q_{\text{IMP},t}^O [P_t^O + \Delta P_t^O + C_{\text{DIST},t}^O + C_{\text{DREF},t}^O + C_{\text{DREF},t}^{\text{CO}_2} + C_{\text{LOSS},t}^O]$, models the costs associated with oil imports to domestic refineries. Price of the imported crude, cost of transporting the imported crude to the domestic refinery system, cost of domestic refining, carbon costs associated with the refining system, and cost of oil distribution losses. The fifth term, $\sum_{t=1}^T \sum_{j=1}^5 q_{\text{SWP},j,t}^P [C_{\text{CT},j,t}^P + C_{\text{DIST},j,t}^P + C_{\text{EMIT},j,t}^{\text{CO}_2} + C_{\text{LOSS},j,t}^P]$, captures the costs associated with delivering refined product into the domestic market by the swap arrangement. Cost of freight from the source market to the Nigerian market, cost of product distribution within domestic market, cost of carbon from utilizing the fuel within the domestic market and oil distribution losses. The sixth term, $\sum_{t=1}^T \sum_{j=1}^5 q_{\text{IMP},j,t}^P [P_{\text{IMP},j,t}^P + C_{\text{CT},j,t}^P +$

$C_{DIST,j,t}^P + C_{EMIT,j,t}^{CO2} + C_{LOSS,j,t}^P$], captures the costs associated with direct petroleum product imports. The free-on-board price of the fuel, cost of freight to domestic market from foreign market, distribution cost of refined products, cost of carbon emissions from utilizing the products and product distribution losses. The seventh component, $\sum_{t=1}^T \sum_{j=1}^5 q_{DOM,j,t}^P [C_{DIST,j,t}^P + C_{EMIT,j,t}^{CO2} + C_{LOSS,j,t}^P]$, represents the costs associated with distributing and utilizing domestically refined products in the domestic market. Costs include the distribution costs with domestic market, carbon

emissions from utilizing the liquid fuel, and associated product distribution loss. $\sum_{t=1}^T \sum_{j=1}^5 q_{EXP,j,t}^P [C_{LOSS,j,t}^P]$ is the eighth component which models the losses associated with refined products exports, and $\sum_{t=1}^T [FC_{DREF,t} + FC_{DIST,t}]$ is the ninth component which represent the fixed costs associated with operating the midstream and downstream infrastructure of domestic refining and pipeline distribution. Subtracting Equation (3) from Equation (2) and rearranging provides the objective function to be maximized.

$$= \sum_{t=1}^T \left[Q_{EXP,t}^O [\alpha_{EXP,t}] + Q_{OFF,t}^O [\alpha_{OFF,t}] + Q_{DOM,t}^O [\alpha_{DOM,t}] + Q_{IMP,t}^O [\alpha_{IMP,t}] + \sum_{j=1}^5 q_{SWP,j,t}^P [\beta_{SWP,j,t}] + \sum_{j=1}^5 q_{IMP,j,t}^P [\beta_{IMP,j,t}] + \sum_{j=1}^5 q_{DOM,j,t}^P [\beta_{DOM,j,t}] + \sum_{j=1}^5 q_{EXP,j,t}^P [\beta_{EXP,j,t}] - FC_{DREF,t} - FC_{DIST,t} \right] \quad (4)$$

Where:

$\alpha_{EXP,t} = P_t^O - C_{PROD,t}^O - C_{PROD,t}^{CO2} - C_{LOSS,t}^O$, the coefficient of the oil export volumes

$\alpha_{OFF,t} = -C_{DT,t}^O - C_{OREF,t}^O - C_{PROD,t}^O - C_{PROD,t}^{CO2} - C_{LOSS,t}^O$, the coefficient of the oil to offshore refining

$\alpha_{DOM,t} = P_t^O - C_{DIST,t}^O - C_{DREF,t}^O - C_{PROD,t}^O - C_{PROD,t}^{CO2} - C_{DREF,t}^{CO2} - C_{LOSS,t}^O$, the coefficient of the oil to domestic refinery system

$\alpha_{IMP,t} = -P_t^O - \Delta P_t^O - C_{DIST,t}^O - C_{DREF,t}^O - C_{DREF,t}^{CO2} - C_{LOSS,t}^O$, the coefficient of the oil imports

$\beta_{SWP,j,t} = P_{DOM,j,t}^P - C_{CT,j,t}^P - C_{DIST,j,t}^P - C_{EMIT,j,t}^{CO2} - C_{LOSS,j,t}^P$, the coefficient of the refined product j swapped

$\beta_{IMP,j,t} = P_{DOM,j,t}^P - P_{IMP,j,t}^P - C_{CT,j,t}^P - C_{DIST,j,t}^P - C_{EMIT,j,t}^{CO2} - C_{LOSS,j,t}^P$, the coefficient of the refined product j imported

$\beta_{DOM,j,t} = P_{DOM,j,t}^P - C_{DIST,j,t}^P - C_{EMIT,j,t}^{CO2} - C_{LOSS,j,t}^P$, the coefficient of the refined product j to domestic market from the domestic refinery system

$\beta_{EXP,j,t} = P_{EXP,j,t}^P - C_{LOSS,j,t}^P$, is the coefficient of the refined product j exported

The coefficients are broadly categorized into two groups – the alpha coefficients (α -coefficients) which relate to crude oil and the beta coefficients (β -coefficients) which relate to the refined products.

The Constraints

The quantity of oil for export, domestic use and offshore refining is constrained by upstream production and expressed as:

$$Q_{EXP}^O + Q_{DOM}^O + Q_{OFF}^O = Q_{PROD}^O \quad (5)$$

The quantity of refined products supplied from the domestic refining system to the domestic market and to the export market is constrained by the crude oil supply to domestic refining system and is expressed as:

$$\sum_{j=1}^5 q_{DOM,j}^P + \sum_{j=1}^5 q_{EXP,j}^P = Q_{IMP}^O + Q_{DOM}^O \quad (6)$$

Quantity of crude oil supplied into domestic refining system is constrained by the Total Domestic Refining Capacity, TDRC, which is expressed as follows:

$$Q_{IMP}^O + Q_{DOM}^O \leq TDRC \quad (7)$$

Quantity of oil sent to the offshore refinery system is constrained by Total Offshore Refining Capacity, TORC, and expressed as follows:

$$Q_{OFF}^O \leq TORC \quad (8)$$

Refined products from offshore refining is constrained by crude allocated to offshore refining and the contractual arrangement adopted in the exchange arrangements (Sayne *et. al.*, 2015) and expressed as

$$\sum_{j=1}^5 q_{SWP,j}^P [P_{IMP,j}^P + F_{SWP,j}^P] = Q_{OFF}^O [P^O - C_{OREF}^O] \quad (9)$$

$F_{SWP,j}^P$ is the “Swap Fee” which is a contractually negotiated fee.

Sum of refined products from domestic refining, offshore refining and independent import is constrained by domestic demand for refined products and expressed as follows:

$$\sum_{j=1}^5 q_{DOM,j}^P + \sum_{j=1}^5 q_{IMP,j}^P + \sum_{j=1}^5 q_{SWP,j}^P = \sum_{j=1}^5 q_{DEM,j}^P \quad (10)$$

The yield constraints from the domestic refining system are non-linear constraints expressed as follows:

$$LB_j \leq \frac{q_{DOM,j}^P + q_{EXP,j}^P}{Q_{IMP}^O + Q_{DOM}^O} \leq UB_j \quad (11)$$

LB_j and UB_j represent the Lower and Upper Bound respectively for the yield of product j from the domestic refining system. Linearizing the above constraint however results in the following pair of constraints for each product yield:

$$q_{DOM,j}^P + q_{EXP,j}^P - UB_j Q_{IMP}^O - UB_j Q_{DOM}^O \leq 0 \quad (12)$$

$$q_{DOM,j}^P + q_{EXP,j}^P - LB_j Q_{IMP}^O - LB_j Q_{DOM}^O \geq 0$$

The non-negative constraints are expressed thus:

$$\begin{aligned} Q_{EXP}^O &\geq 0 & q_{DOM,j}^P &\geq 0 \\ Q_{OFF}^O &\geq 0 & q_{IMP,j}^P &\geq 0 \\ Q_{DOM}^O &\geq 0 & q_{EXP,j}^P &\geq 0 \\ Q_{IMP}^O &\geq 0 & \text{For } j = 1 \dots 5 & \end{aligned} \quad (13)$$

$$q_{SWP,j}^P \geq 0$$

The Input Data

Data input to the optimization framework is obtained from varied sources which are summarized in

Table 1. Forecast of these inputs are prepared to be fed into the optimization model.

Table 1: Parameters and sources

S/N	Parameters	Data Source
1	Oil price, P^O	IEA NZE2050 scenario
2	Oil production, Q_{PROD}^O	IEA NZE2050 scenario
3	Cost of upstream oil production, C_{PROD}^O	Regression model (Gbakon, <i>et. al.</i> 2021)
4	Dirty Tanker Freight, C_{DT}^O	Argus Media
5	Oil pipeline distribution costs, C_{DIST}^O	Sayne <i>et. al.</i> 2015

S/N	Parameters	Data Source
6	Price differential of imported oil, ΔP^O	Argus Media
7	Variable cost of domestic refining, C_{DREF}^O	Sayne <i>et. al.</i> 2015
8	Fixed cost domestic refining, FC_{DREF}	NNPC F&O reports, Reuters
9	Offshore refining processing fee, C_{OREF}^O	Sayne <i>et. al.</i> 2015
10	Clean Tanker Freight, C_{CT}^P	Argus Media
11	Domestic demand of refined products, $q_{DEM,j}^P$	Woodmac
12	Product yields: domestic refineries LB_j, UB_j	NNPC ASB, EIA Dangote refinery
13	Domestic prices of refined products, $P_{DOM,j}^P$	PPPRA, Platts

For the projection of product demand, we utilize the forecasts provided by Wood MacKenzie from their Sub-Saharan Africa product markets 2021 outlook to 2050 (2021). Abdullahi *et. al.* (2016) had developed forecasts of petroleum product

consumption in Nigeria which were consistent with the forecasts provided by Wood McKenzie at the time. Consequently, we adopt the Wood McKenzie petroleum product demand forecasts shown in Table 2.

Table 2: Nigeria petroleum product demand forecast (Woodmac)

	MMbbls					Totals
	LPG	Gasoline	DPK	AGO	Fuel Oil	
2021	5.79	118.24	23.66	31.79	8.06	187.54
2025	6.37	135.13	25.01	35.39	6.71	208.61
2030	7.18	153.06	27.19	42.93	7.03	237.39
2035	8.08	171.59	28.69	50.59	6.77	265.72
2040	9.08	188.89	30.73	60.53	6.82	296.05

2.3 The three scenarios

Table 3 form the basis on which to determine the optimal allocation of crude oil. The “Business as usual” scenario is intended to represent the continuing state of the energy and

The scenarios in policy environment. The “Stated Policy” scenario captures the intentions of government as expressed in policy statements or acts. The “Energy Transition” scenario localizes attributes contained in IEA NZE2050 policy for Nigeria to be consistent with the goal of Net Zero Emissions by 2050.

Table 3: Description of scenarios

Dimension	Business As Usual	Stated Policy	Energy Transition
1 Oil Production	Production continues decline at historical rate of 5% (between 2010 and 2020)	Production increases by 1.20% pa. This as per IEA Stated Policy scenario for OPEC.	Production decline by 3.30% pa as per the IEA NZE2050
2 Oil Price Profile	Oil Prices (RT2019) increase from \$69/bbl (2021) to \$80/bbl by 2040. This is the EIA Reference oil price case in the Annual Energy Outlook 2022	Oil Prices (RT2019) increase from \$69/bbl (2021) to \$80/bbl by 2040. This is the EIA Reference oil price case in the Annual Energy Outlook 2022	Oil price (RT2019) declines as per IEA NZE2050 scenario from \$37/bbl (2021) to \$29/bbl (2040)

Dimension	Business As Usual	Stated Policy	Energy Transition
3 CO ₂ price	No CO ₂ price is contemplated in this scenario as there has been no government policy in this area	No CO ₂ price is contemplated in this scenario as there has been no government policy in this area	CO ₂ price (RT2019) increases linearly as per IEA NZE2050 scenario from \$3/t (2025) to \$35/t (2040)
4 PMS price subsidy	Subsidy exists between 2021 – 2029. Subsidy is removed 2030+	Subsidy exists between 2021 – 2024. Subsidy is removed 2025+	Subsidy exists between 2021 – 2023. Subsidy is removed from 2024+
5 Domestic Refining Capacity Build-up	DORC (start 2023) + PHRC (start 2025)	DORC (start 2023) +PHRC (start 2025) +WRPC(start 2027)+KRPC(start 2029)	DORC (start 2023) + PHRC (start 2025)

3. Results and discussion

The outcome from the optimal oil allocation under the three different scenarios is presented in this section. Under each of the three scenarios, two key metrics are evaluated – oil exports and net benefit.

3.1 Business as usual scenario

Oil Exports

With both oil production declining and product demand increasing, the optimal pathway is for oil exports to decrease from 612 MMbbls in 2021 to ~ 70 MMbbls in 2030. By 2035, there are no oil volumes for export, instead ~ 16 MMbbls of oil is imported to be refined by the domestic refining system. Oil imports increase to 83 MMbbls by 2040. Domestic utilization of crude oil is maintained at ~ 314 MMbbls through the period from 2025 to 2040 keeping the domestic refining system at capacity.

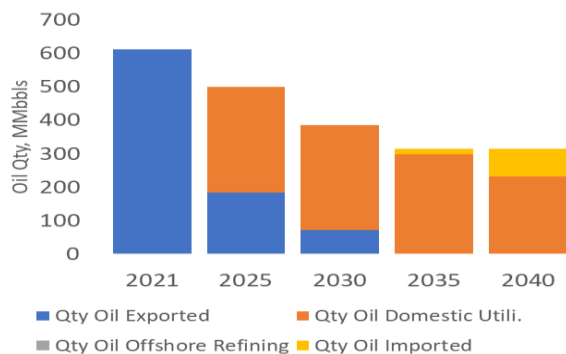


Fig. 2: Optimal allocation of Nigeria's oil production under "business as usual" scenario

Net benefits

In undiscounted terms, the sum of net benefits over the period under the "Business as Usual" scenario amounts to \$423 billion. As previously stated, net benefit is the culmination of several factors relating to oil supply, product exports, imports, and other fixed costs. These contributions to the net benefit are illustrated in Fig. 2.

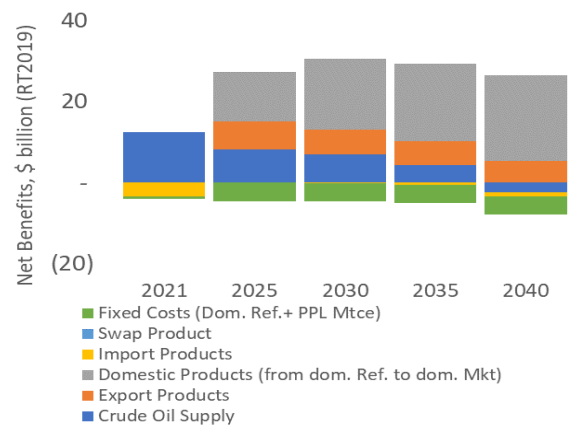


Fig. 3: Distribution of optimal net benefits under "business as usual" scenario

Net benefit under "Business as Usual" is driven largely by net value from domestic product supply from domestic refining which increases from \$12 billion (2025) to \$21 billion (2040) compared to net value from crude supply which decreases from \$12.5 billion (2021) to a loss of \$2.3 billion in 2040. Net value from product exports declines from \$6.8 billion (2025) to \$5.3 billion (2040). Under the

“Business as Usual” scenario, product supply to the domestic market from domestic refining as well as product export are the leading drivers of value. Value from crude supply, though initially higher than product export, declines through time such to be lower than value derived from product export.

3.2 The stated policy scenario

Oil exports

With both oil production and product demand increasing under this scenario, we find that the optimal pathway sees a reduction in oil exports from 765 MMbbls in 2022 to ~ 400 MMbbls in 2029 from which it increases to 476 MMbbls by 2040. Domestic utilization of crude oil produced is maintained at ~ 400 MMbbls through the period from 2029 to 2040 which explains the increasing amounts of oil exports occasioned by upstream oil production increases. This trend is detailed in Fig. 4. It is also worthy to note that under this scenario, there is no oil dedicated to offshore refining and none is imported to augment domestic use.

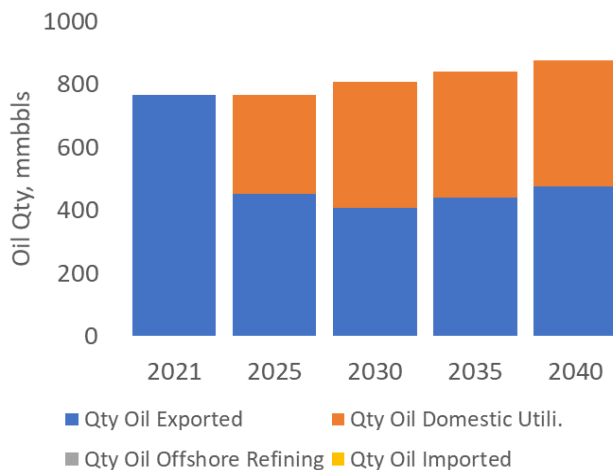


Fig. 4: Optimal Allocation of Nigeria's oil production under the “stated policy” scenario

Net benefits

The sum of net benefits over the period under the “Stated Policy” scenario amounts to \$718 billion in undiscounted terms. The factors contributing to the net benefit is illustrated in Fig. 5 – factors relating to oil supply, product exports, imports and other fixed costs.

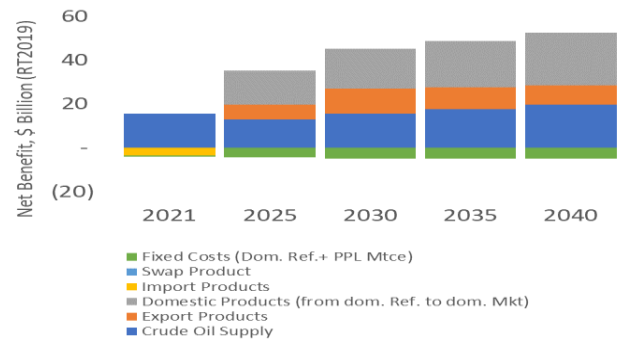


Fig. 5: Distribution of optimal net benefits under “stated policy” scenario

Contribution of net value from domestic product sales exceeds that from crude oil supplies. Net value from domestic product supply from domestic refining increases from \$10 billion (2023) to \$24 billion (2040) compared to net value from crude supply which increases from \$10.8 billion (2023) to \$19.4 billion (2040). Net value from product exports rises from \$3.4 billion (2023) to \$9 billion (2040). Under the Stated Policy scenario, product supply to the domestic market from domestic refining as well as crude supplies (both to export and domestic refining system) are both important drivers of value.

3.3 The energy transition scenario

Oil exports

With increasing petroleum product demand, the optimal pathway under this scenario is a decline in oil exports from 623 MMbbls in 2022 to ~ 19 MMbbls in 2040 as per Fig. 6. This is consistent with the oil production decline expected under this scenario. Conversely, domestic utilization of crude oil produced is maintained at 314 MMbbls through the period from 2025 to 2040. It is also worthy to note that there is no oil dedicated to offshore refining.

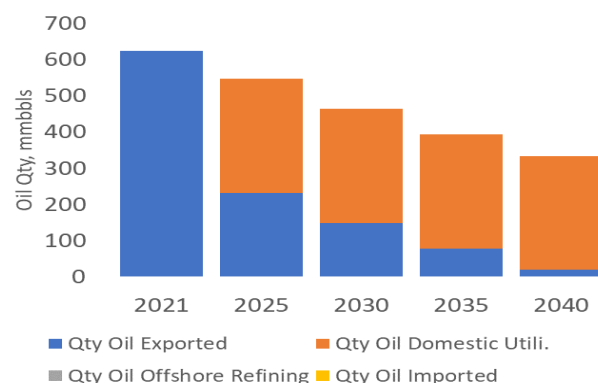


Fig. 6: Optimal allocation of Nigeria's oil production under energy transition

Net benefits

Recall that the objective of the optimization model is the maximization of net benefits indicated as the difference in revenue inflows and cost outflows as per the Reference Energy System (see **Error! Reference source not found.**). In undiscounted terms, the sum of net benefits over the period amounts to \$192 billion. The contribution of oil supply, product exports, imports and other fixed costs to the net benefit is illustrated in Fig. 7.

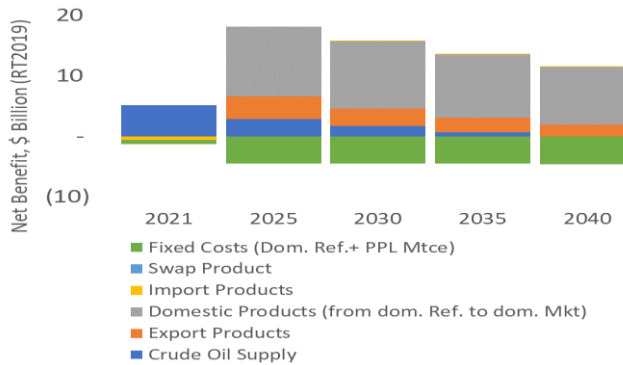


Fig. 7: Distribution of optimal net benefits under “energy transition” scenario

Table 4 shows the distributions used for the input data.

Table 4: Probability distributions for Monte Carlo simulation

Name of Variable	@Risk Function	Remarks
Oil Production Growth	RiskUniform(-0.05,0.02,RiskStatic(-0.033))	Uniform distribution of growth between -5% and 2%
Oil Price Growth	RiskUniform(-0.04,0.01,RiskStatic(-0.02))	Uniform distribution of growth between -4% and 1%
Duration of Subsidy (yrs)	RiskDiscrete(D499:D517,E499:E517,RiskStatic(5))	Discrete distribution
Dangote Refinery Start: # Yrs from 2021	RiskDiscrete(D520:D524,E520:E524,RiskStatic(2))	Discrete distribution
Dangote Refinery CapEx	RiskTriang(18000,19000,24000,RiskStatic(19000))	Triangular distribution with minimum CAPEX \$18B, max of \$24B and most likely of \$19B
PHRC Start: # Yrs from 2021	RiskDiscrete(D527:D531,E527:E531,RiskStatic(4))	Discrete distribution
PHRC CapEx (Rehab)	RiskUniform(1500,2000,RiskStatic(1500))	Uniform distribution between \$1.5B and \$2B
WRPC Start: # Yrs from 2021	RiskIntUniform(6,10,RiskStatic(6))	Integer Uniform distribution between 6 and 10
WRPC CapEx (Rehab)	RiskUniform(1200,1500,RiskStatic(1200))	Uniform distribution between \$1.2B and \$1.5B
KRPC Start: # Yrs from 2021	RiskIntUniform(8,12,RiskStatic(8))	Integer Uniform distribution between 8 and 12
KRPC CapEx (Rehab)	RiskUniform(1200,1500,RiskStatic(1200))	Uniform distribution between \$1.2B and \$1.5B

The greatest contribution to the net benefits comes from domestic sales of products from the domestic refineries. Between \$8 billion and \$11.5 billion annually is attributable to product supply to the domestic market from the domestic refining system. Meanwhile, the contribution from product exports declines from \$3.8 billion (2025) to \$1.9 billion (2040). The net value from crude oil sales (both to export and to domestic refineries) declines from \$2.9 billion (2025) to less than \$1 billion in 2035. These statistics on the net value contributions to the system net benefits highlight the importance of product supply to the domestic market from domestic refining and the diminishing import of the value from oil supply (both to export and domestic refining system).

3.4 Monte Carlo simulation

The results of stochastic analysis of the optimization model are presented in this section. By defining inputs to the model as random variables with probability density functions, the net benefit is obtained as a distribution function that enables discussion of outcomes as probabilities.

Name of Variable	@Risk Function	Remarks
Pipeline Start: # Yrs from 2021	RiskIntUniform(4,8,RiskStatic(4))	Integer Uniform Distribution between 4 and 8 years
Pipeline and Evacuation (Rehab)	RiskUniform(3500,6000,RiskStatic(4000))	Uniform distribution between \$3.5B and \$6B

The uncertainty in product demand forecast is modeled as probabilistic time series within the @RISK software using the Time Series Fit functionality. The time series model is obtained by fitting historical trend to different models and the fit which results in the least Akaike Information Criteria (AIC) is selected. This best fit model is used to then project the demand series into the future. For every iteration run during the stochastic simulation using @RISK, a different future product demand profile is chosen. The future demand profile will fall within specified confidence interval.

Table 5 shows time series models for demand of the five products. LPG historical demand is modelled as a Brownian Motion with Mean Reversion Jump Diffusion process. This is then the basis for making probabilistic time series forecast. The symbols in the function $BMMRJD(\mu, \sigma, \alpha, \lambda, \mu_j, \sigma_j, Y_0)$ are defined as follows: μ is drift, σ is volatility, α is the speed of reversion, λ is the jump rate, μ_j is the jump size mean, σ_j is the jump size standard deviation, and Y_0 is the value of the data feed at time 0.

The historical gasoline demand is modelled as Auto – Regressive Conditional Heteroskedasticity of a first order (ARCH1) process. Mean gasoline demand is forecast to grow to 200 MMbbls by year 2040. The symbols in the function $ARCH1(\mu, \omega, \alpha_1, Y_0)$ are defined as follows: μ is the mean, ω is the volatility parameter, α_1 is the error coefficient, and Y_0 is the value of data feed at time 0. The historical DPK demand is modelled as a

Moving Average of first order (MA1) process. The mean DPK demand is thus forecast to be ~13 MMbbls by 2040. The symbols in the function $MA(\mu, \sigma, b_1, O)$ are defined as follows: μ is the mean, σ is the volatility parameter, b_1 is the moving average coefficient, and ε_0 is the initial error term. The historical demand for AGO is modelled as an Auto – Regressive Conditional Heteroskedasticity of a first order (ARCH1) process and the mean AGO demand by 2040 is forecast to be ~ 43 MMbbls. Mean fuel oil (FO) demand by 2040 is 10 MMbbls and its historical demand is modelled as an Auto – Regressive Conditional Heteroskedasticity of a first order (ARCH1) process. Both AGO and FO are modelled as ARCH1 processes, just like gasoline, which symbols have already been defined.

While the product demands are all modelled as probabilistic time series, the different model-types are based on a historical fit which results in the least Akaike Information Criteria (AIC). The best-fit model is selected from a set of thirteen candidate models in the @RISK software using the AIC metric. The AIC metric estimates the models’ prediction error and thus establishes their relative quality (Stoica & Selen, 2004). Given that information loss results when statistical models are used to represent the data modelled, the AIC selects the most efficient model – the one that retains the most information with the most parsimonious representation. For the Monte Carlo Simulation, the “Net Benefit” metric is chosen to analyze the outcome of the optimization.

Table 5: Summary of product demand time series models selected by AIC

S/N	Product	Model Process	Specification of Arguments
1	LPG	Brownian Motion with Mean Reversion Jump Diffusion $BMMRJD(\mu, \sigma, \alpha, \lambda, \mu_j, \sigma_j, Y_0)$	$\mu = 8897$ $\sigma = 0.75047$ $\alpha = 8.0923E-05$ $\lambda = 1.7304E-05$ $\mu_j = 4022.4$ $\sigma_j = 11.482$ $Y_0 = 11.3$
2	PMS	Auto – Regressive Conditional Heteroskedasticity of a first order. $ARCH1(\mu, \omega, \alpha_1, Y_0)$	$\mu = 3.9062$ $\omega = 18.484$ $\alpha_1 = 0.28786$

			$Y_0 = 7.665$
3	DPK	Moving Average of first order $MA(\mu, \sigma, b_1, \varepsilon_0)$	$\mu = 13.197$ $\sigma = 2.144$ $b_1 = 0.91556$ $\varepsilon_0 = -4.7226$
4	AGO	Auto – Regressive Conditional Heteroskedasticity of a first order $ARCH1(\mu, \omega, \alpha_1, Y_0)$	$\mu = 0.60544$ $\omega = 3.6923$ $\alpha_1 = 0.35017$ $Y_0 = -1.095$
5	FO	Auto – Regressive Conditional Heteroskedasticity of a first order $ARCH1(\mu, \omega, \alpha_1, Y_0)$	$\mu = 10.014$ $\omega = 6.5165$ $\alpha_1 = 0.014039$ $Y_0 = 9.475$

The probability distribution of the Net Benefit is shown in Fig. 8. Based on the distribution, there is a 95% likelihood that the net benefit will exceed ~\$360 billion. Note that this is ~ 2X (or > \$170 billion higher than) the deterministic value obtained under the “Energy Transition” scenario. The import of this observation is that the likelihood that the net benefit is less than that obtained under “Energy Transition” scenario is <5%. Similarly, the

likelihood that net benefit exceeds \$187 billion is 100%. Furthermore, there is a 0% likelihood that net benefits exceed \$600 billion, thus making this the maximum value to be expected. Situating this against the \$718 billion obtained under the “Stated Policy”, it can be said that there is a 0% likelihood that the benefits expected under the “Stated Policy” scenario will materialize.

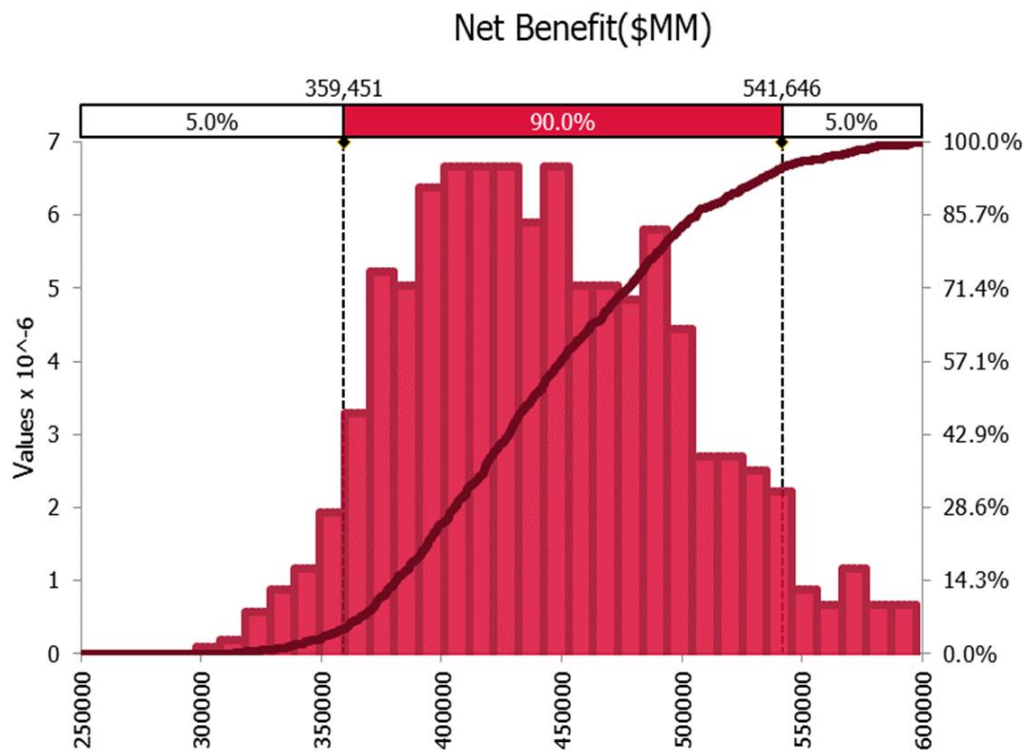


Fig. 8: Probability distribution of the lifecycle net benefit

Table 6: Comparison of net-benefit under scenarios vs likelihood of occurrence

Scenario	Net Benefit (\$ billion)	Likelihood P (X<Net Benefit)
Business as Usual	423	40%
Stated Policy	718	100%

4. Conclusion

A mathematical program has been developed for optimal end-use allocation of nationally produced crude oil and refined products under three main scenarios – the “Business as Usual,” “Stated Policy,” and “Energy Transition” scenarios. Stochastic analysis has also been conducted. Under the “Business as Usual” scenario, a net benefit of \$423 billion (RT2019) is realised over the period 2021 to 2040. This value is driven mostly by refined product supply from the domestic refineries to the domestic market. Export of refined products also plays a significant role in the net value added to the net benefits. However, the declining importance of crude oil supplies serves to highlight the diminishing inflows of foreign exchange while the domestic supply of refined products from the local refining system would imply that more inflows in the local currency begin to occur. Additionally, less expenditure on product imports would signal lower spending in foreign exchange. Net benefits total \$718 billion (RT2019) under the “Stated Policy” scenario, driven largely by net value from crude oil supplies and domestic supply of refined products from local refinery system. Net value from refined product export is significant and reaches ~ \$4 billion (RT2019) by 2040. Under this policy, there is no room for product imports and instead by 2030, a peak of 166 MMbbls of products – consisting mostly of kerosene and diesel – is to be exported, which is 42% of domestic refinery production. Product export declines to 112 MMbbls by 2040 representing 28% of refinery production. \$192 billion (RT2019) is the Net benefit resulting under the “Energy Transition” scenario. This value is driven by contribution from the domestic product supply from local refining system. Some inflow is to be expected from product exports. The value from crude oil supply diminishes to 0 after 2035 and this is on account of the decline in oil production consistent with the net zero prescription. In this scenario, 16% of product demand is expected to be imported driven by gasoline demand. However, around 20% of refinery production will be exported by 2040 driven by the distillates and fuel oil. Stochastic analysis shows that there is a ~50% likelihood that the net benefit associated with the “Business as Usual” scenario is exceeded. The net benefit captured under the “Energy Transition” and “Stated Policy” scenarios shows a 0% likelihood of materializing. A common theme across the scenarios evaluated is that net benefit is

expected to be driven by the supply of refined products to the domestic market from domestic refining while value from crude oil supply recedes (except in the “Stated Policy” scenario). Thus, to achieve the robust optimization of Nigeria’s petroleum value chain, the development of Midstream and Downstream infrastructure is inevitable. For Nigeria as an oil exporting country the addition of the 650 Mbbls/day Dangote refinery (20% of Africa’s current refinery capacity) has significant fiscal and geopolitical consequence when juxtaposed against whatever view may be taken of future oil production. Additionally, the “Energy Transition” scenario delivers the least net benefit of all three scenarios modelled. This highlights the economic import of the energy transition to Nigeria. The “Energy Transition” scenario delivers \$231 billion less value than the “Business as Usual” scenario and is suggestive of the fiscal challenges posed by the Energy Transition to Nigeria. Policy makers in weighing the possibility of this scenario must contemplate alternatives to improve the woeful fiscal position promised under this scenario.

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