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Prediction of Compressive Strength of Periwinkle Shell Aggregate Concrete Mixes

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Abstract

In this paper, a mathematical model is developed to predict the compressive strength of periwinkle shell aggregate concrete using Osadebe's regression approach. The results obtained from the model coincided with those obtained from experiment. The model was tested for adequacy using F-statistics at 5% level of significance and was found to be adequate. A computer program coded in BASIC language was written based on the formulated model for easy prediction of compressive strength of periwinkle shell aggregate concrete. The written computer program based on the formulated model, can predict the appropriate mix ratios corresponding to any desired compressive strength value without waste of time.

Keywords:*Mathematical model, compressive strength, periwinkle shell aggregate concrete, mix ratios, desired compressive strength value*

1. Introduction

Periwinkle shells are abundant in the swampy mud and river banks of the Niger Delta area of Nigeria. They are abundant in the South-South geo-political zone of Nigeria. Their lengths vary from 31.8mm to 63.96mm with an average diameter of about 16.9mm near one end and tapering to a point at the other end. Coarse aggregate occupies about 75% of total concrete volume and is rated as the second most important component of concrete after cement (Gupta, 2013). The high cost of granite is one of the factors that have hindered most citizens of Nigeria from affording their own shelters. Consequently, any material that can replace granite as coarse aggregates in concrete production and is much cheaper would go a long way to reducing the cost of concrete production cost of concrete production in Nigeria. Recently, periwinkle shells have been used to replace granites in concrete production (Beredugo, 1984; Awakara; 1975; Egamana, 2006; Osadebe and Ibearuegbulem, 2009). Efforts are therefore, being directed towards the use of local materials that can replace granite in concrete production without adverse effects on the strength properties of concrete. The value of compressive strength of concrete depends on the mix ratio of component materials (Osadebe et al, 2007; Osadebe and Nwakonobi, 2007; Obam and Osadebe, 2006; Ndububa and Osadebe, 2007).

In this paper, the Osadebe's regression technique is used to predict the compressive strength of periwinkle shell aggregate concrete. A computer program is written based on the formulated model to select the appropriate mix ratios corresponding to desired values of compressive strength of periwinkle shell aggregate concrete.

2. Development of strength model

According to Obam and Osadebe (2006), the response function can be approximated by Taylor series as follows:

$$f(Z) = f(Z^{(0)}) + \sum_{i=1}^{4} \frac{\partial f(Z^{(0)})}{\partial Z_i}(Z_i - Z_i^{(0)}) + \frac{1}{2}!\sum_{i=1}^{3}\sum_{j=1}^{4} \frac{\partial^2 f(Z^{(0)})}{\partial Z_i \partial Z_j}(Z_i - Z_i^{(0)})(Z_j - Z_j^{(0)}) + \frac{1}{2}!\sum \frac{\partial^2 f(Z^{(0)})}{\partial Z_i^{\,2}}(Z_i - Z_i^{(0)})^2 + \ldots \quad (1)$$

Consider S_i and Z_i to be the actual proportions and fractional portions of the mixture components respectively. If the total quantity of the concrete mixture is S, then for a concrete mixture consisting of 4 components,

$$S_1 + S_2 + S_3 + S_4 = S \quad (2)$$

Dividing both sides of Equation (2) yields:

$$S_1/S + S_2/S + S_3/S + S_4/S = 1 \quad (3)$$

Consequently,

$$S_i/S = Z_i, (i = 1, 2, 3, 4) \quad (4)$$

Equation (3) now transforms to:

$$Z_1 + Z_2 + Z_3 + Z_4 = 1 \quad (5)$$

where Z_1, Z_2, Z_3, Z_4 = proportion of water, cement, sand and granite respectively

The vector of elements $Z_i\,(Z_1, Z_2, Z_3, Z_4)$ are subject to the constraint imposed of Equation (5) and for each Z_i,

$$Z_i > 0 \quad (6)$$

$Z^{(0)}$ = 0 for the purpose of convenience.

$$\Rightarrow Z_1^{(0)} = 0, Z_2^{(0)} = 0, Z_3^{(0)}, Z_4^{(0)} = 0 \quad (7)$$

Let:

$$b_0 = f(0), \quad b_i = \frac{\partial f(0)}{\partial z_i}, \quad b_{ij} = \frac{\partial^2 f(0)}{\partial z_i \partial z_j}, \quad and \quad b_{ii} = \frac{\partial^2 f(0)}{\partial z_i^2}$$

After substitution, Equation (1) becomes:

$$f(z) = b_o + \sum_{i=1}^{4} b_i z_i + \sum_{i=1}^{3}\sum_{i=1}^{4} b_{ij} z_i z_j + \sum_{i=1}^{4} b_{ii} z_i^2 + \ldots (8)$$

Multiplication of Equation (5) by b_0 yields:

$$b_0 = b_0 Z_1 + b_0 Z_2 + b_0 Z_3 + b_0 Z_4 + b_0 Z_5 \; (9)$$

Multiplying Equation (5) by Z_i and rearranging the same Equation yields expressions for $Z_i^{\,2}$ as follows:

$$Z_1^2 = Z_1 - Z_1 Z_2 - Z_1 Z_3 - Z_1 Z_{45} \quad (10)$$

$$Z_2^2 = Z_2 - Z_1 Z_2 - Z_2 Z_3 - Z_2 Z_4 \quad (11)$$

$$Z_3^2 = Z_3 - Z_1 Z_3 - Z_2 Z_3 - Z_3 Z_4 \quad (12)$$

$$Z_4^2 = Z_4 - Z_1 Z_4 - Z_2 Z_4 - Z_4 Z_3 \quad (13)$$

Substitution of Equations (10) to (13) into Equation (8) and after factorizing and setting $f(z) = y$ transforms Equation (8) to:

$$y = (b_0 + b_1 + b_{11})Z_1 + (b_0 + b_2 + b_{22})Z_2 + (b_0 + b_3 + b_{33})Z_3 + (b_0 + b_4 + b_{44})Z_4 + (b_{12} - b_{11} - b_{22})Z_1 Z_2 + (b_{13} - b_{11} - b_{33})Z_1 Z_3 + (b_{14} - b_{11} - b_{44})Z_1 Z_4 + (b_{23} - b_{22} - b_{33})Z_2 Z_3 + (b_{24} - b_{22} - b_{44})Z_2 Z_4 + (b_{34} - b_{33} - b_{44})Z_3 Z_4) \quad (14)$$

Let: $\alpha_i = b_0 + b_i + b_{ii}$, $\alpha_{ij} = b_{ij} - b_{ii} - b_{jj}$ (15)

From Equation (15), Equation (14) transforms to:

$$y = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3 + \alpha_4 Z_4 + \alpha_5 Z_5 + \alpha_{12} Z_1 Z_2 + \alpha_{13} Z_1 Z_3 + \alpha_{14} Z_1 Z_4 + \alpha_{15} Z_1 Z_5 + \alpha_{23} Z_2 Z_3 + \alpha_{24} Z_2 Z_4 + \alpha_{25} Z_2 Z_5 + \alpha_{34} Z_3 Z_4 + \alpha_{35} Z_3 Z_5 + \alpha_{45} Z_4 Z_5 \quad (16)$$

where y = compressive strength at a particular mix ratio, Z_i, Z_j, Z_{ij} = predictor variables and α_i, α_{ij} = constant coefficients of the model.

2.1. The coefficients of the model

At nth experimental point, $y^{(n)}$ the corresponding vector of the predictor variables is given by:

$$Z^{(n)} = [Z_1^{(n)}, Z_2^{(n)}, Z_3^{(n)}, Z_4^{(n)}, Z_5^{(n)}] \quad (17)$$

And the corresponding response function, Y $^{(n)}$ in terms of the predictors' variables $Z_i^{(n)}$ and constant coefficients, α_i *and* α_{ij} is given by:

$$y^{(n)} = \sum_{i=1}^{5} \alpha_i Z_i^{(n)} + \sum_{1 \le i \le j}^{5} \alpha_{ij} Z_i^{(n)} Z_j^{(n)} \quad (18)$$

where $1 \le i \le j \le 4 \; and \; n = 1, 2, 3, ..., 10$

Equation (18) can be written in matrix form as:

$$[y^{(n)}] = [Z^{(n)}][\alpha] \quad (19)$$

The constant coefficients, α_i in Equation (19) are obtained from the known values of $Y^{(n)}$ *and* $Z^{(n)}$.

From equation (19),

$$[\alpha] = [Z^{(n)}]^{-1}[y^{(n)}] \quad (20)$$

The actual proportions $S_i^{(n)}$ and their corresponding fractional portions $Z_i^{(n)}$ are given in Table 1. The $Z^{(n)}$ matrix and $Z^{(n)}$ matrix inverse are obtained from the known values of fractional parts, $Z^{(n)}$. The values of $Y^{(n)}$ matrix are obtained from experiments.

Let the actual and pseudo components be denoted by S_i and X_i respectively. According to Osadebe, the relationship between S_i and X_i is given by:

$$S = AX \quad (21)$$

where A = coefficient of the actual mix ratios, S = actual mix ratio and X = pseudo mix ratio.

Using Equation (21), the actual components for trial and control mixes are obtained. Similarly, the inverse transformation from pseudo to actual components (S_i) according to Osadebe is given by:

$$X = BS \quad (22)$$

where B is the inverse of S matrix.

Scheffe (1958) established the mathematical relationship between the required number of experimental points, the number of component materials and the degree of polynomial as:

$$N = \frac{(q + n - 1)!}{n!(q - 1)!} \quad (23)$$

where n, q = degree of polynomial and number of concrete components respectively

From Equation (23), the number of experimental points for a 4-component concrete mixture is:

$$\frac{(4 + 2 - 1)!}{2!(4 - 1)!} = 10$$

3. Materials and method

The periwinkle shells used for this study were obtained from periwinkle heaps at Okrika waterside, by Hospital Road, Port Harcourt, Rivers State, Nigeria. The length varied from 31.8mm to 63.96mm and diameter of about 16.9mm near one end and tapering to a point at the other end. The periwinkle shell aggregates were thoroughly cleaned with water to remove dirt and then sundried before usage. The fine aggregate was obtained from OPi River, Nsukka in Enugu State, Nigeria. It was prepared to BS 1017: Parts 1 and 2 (1983) and BS 882 (1992). The grading was carried out to BS 812:103 (1975). The sand belongs to grading zone 2. The cement was Dangote cement and was obtained from one of the cement shops in Nsukka market. The water was fresh and had no organic materials.

3.1. Compressive strength test

Concrete cube specimens of size 150mm x 150mm x 150mm were made and tested for compressive strength at 28 days. The cube specimens were prepared by filling each moulds in three layers and compacted in accordance with the requirements of BS1377: Part 4 (1990).

The cubes were removed from moulds after 24 hours of casting after which they were taken to a curing tank to be cured for 28 days. The cubes were then weighed and tested in compression using a compression machine. The maximum load at which the specimen failed was recorded. Two replicates where produced per mix ratio and this gave a total of 32 cubes. The compressive strength, (f_c) of periwinkle shell aggregate concrete was determined from:

$$f_c = \frac{Maximum\,load\,at\,failure}{Area\,of\,cross-section\,of\,cube}\ N/mm^2 \quad (24)$$

Table 1: Actual mix ratio of components and their corresponding fractions

S/No	S1	S2	S3	S4	Z1	Z2	Z3	Z4
1	0.6	1	1.5	3	0.098361	0.16393	0.245902	0.491803
2	0.5	1	1	2	0.111111	0.22222	0.222222	0.444444
3	0.55	1	2	3	0.083969	0.15267	0.305344	0.458015
4	0.65	1	3	6	0.061033	0.0939	0.28169	0.56338
5	0.55	1	1.25	2.5	0.103774	0.18868	0.235849	0.471698
6	0.575	1	1.75	3	0.090909	0.1581	0.27668	0.474308
7	0.625	1	2.25	4.5	0.074627	0.1194	0.268657	0.537313
8	0.525	1	1.5	2.5	0.095023	0.181	0.271493	0.452489
9	0.575	1	2	4	0.075908	0.13201	0.264026	0.528053
10	0.6	1	2.5	5.4	0.063158	0.10526	0.263158	0.568421
Control points								
11	0.576	1	1.875	3.75	0.079989	0.13887	0.260381	0.520761
12	0.538	1	1.375	2.5	0.09939	0.18474	0.254018	0.461851
13	0.563	1	1.625	3.25	0.079989	0.13887	0.260381	0.520761
14	0.563	1	1.5	2.75	0.09939	0.18474	0.254018	0.461851
15	0.588	1	1.75	3.5	0.08745	0.15533	0.252408	0.504815
16	0.55	1	1.625	2.75	0.096852	0.17203	0.258042	0.473078

S_1 = Actual proportion of water, S_2 = Actual proportion of cement, S_3 = Actual proportion of sand, and S_4 = Actual proportion of periwinkle shell

Table 2: Zn matrix derived from Table 1

Z1	Z2	Z3	Z4	Z1Z2	Z1Z3	Z1Z4	Z2Z3	Z2Z4	Z3Z4
0.098361	0.163934	0.245902	0.491803	0.016125	0.024187	0.048374	0.040312	0.080623	0.120935
0.111111	0.222222	0.222222	0.444444	0.024691	0.049383	0.049383	0.049383	0.098765	0.098765
0.083969	0.152672	0.305344	0.458015	0.01282	0.02564	0.038459	0.046617	0.069926	0.139852
0.061033	0.093897	0.28169	0.56338	0.005731	0.017192	0.034385	0.02645	0.0529	0.158699
0.103774	0.188679	0.235849	0.471698	0.01958	0.024475	0.04895	0.0445	0.089	0.11125
0.090909	0.158103	0.27668	0.474308	0.014373	0.025153	0.043119	0.043744	0.074989	0.131232
0.074627	0.119403	0.268657	0.537313	0.008911	0.020049	0.040098	0.032078	0.064157	0.144353
0.095023	0.180995	0.271493	0.452489	0.017199	0.025798	0.042997	0.049139	0.081898	0.122848
0.075908	0.132013	0.264026	0.528053	0.010021	0.020042	0.040083	0.034855	0.06971	0.13942
0.063158	0.105263	0.263158	0.568421	0.006648	0.01662	0.0359	0.027701	0.059834	0.149584

Table 3: Zn matrix inverse

19051.52	5832	-7E-11	3629.52	-21573.1	2.09E-11	-17956	6.08E-11	11017.08	-8.3E-11
3694.209	1630.53	-1.6E-11	190.5498	-5042.72	4.61E-12	-2289.39	1.34E-11	1817.818	-1.8E-11
-32.4013	45.98308	283.8165	-208.697	109.9399	-172.332	246.895	-225.42	-220.342	173.5577
-21.3242	16.66731	41.2524	-102.08	6.050154	30.77356	84.16875	-46.9625	-137.714	130.1683
-39534.9	-13634.7	1.55E-10	-5544.09	47649.63	-4.6E-11	33263.49	-1.3E-10	-22199.4	1.84E-10
-20414.8	-4834.95	-877.851	-5144.84	20405.27	4849.913	18382.45	-2028.78	-11898.4	1562.019
-19761.2	-6564.58	255.7649	-3391.33	23264.57	-2369.56	20004.11	1174.062	-11613.8	-997.957
-3109.15	-2256.72	-191.411	580.7232	5384.983	-1871.03	2064.94	1634.295	-1542.39	-694.231
-3319.62	-1254.22	-87.4551	-197.355	4212.809	990.9086	1228.864	-657.475	-1046.62	130.1683
166.157	-115.737	-549.482	691.8772	-210.027	166.1772	-757.519	479.0175	780.3765	-650.841

4. Results and discussion

The results of the compressive strength obtained from both the trial and control mixes are as given in Table 4. The compressive strength of each cube was determined using equation (24).From Table 4, the coefficients of Equation (20) are obtained as follows:

$\alpha_1 = 64391.526$, $\alpha_2 = 10993.8859$, $\alpha_3 = -1524.289$, $\alpha_4 = -126.583$, $\alpha_{12} = -126652.636$, $\alpha_{13} = -62811.815$, $\alpha_{14} = -70057.908$, $\alpha_{23} = -8380.082$, $\alpha_{24} = -9607.159$, $\alpha_{34} = 3165.693$

Substitution of the obtained values of coefficients into Equation (16) yields:

$$y = 64391.526 Z_1 + 10993.8859 Z_2 - 1524.289 Z_3 - 126.583 Z_4 - 126652.636 Z_1Z_2 - 62811.815 Z_1Z_3 - 70057.908 Z_1Z_4 - 8380.082 Z_2Z_3 - 9607.159 Z_2Z_4 + 3165.693 Z_3Z_4 \quad (25)$$

Equation (25) is the mathematical model for the prediction of compressive strength of periwinkle shell aggregate concrete based on 28 days' strength.

Table 4: Results of compressive strength based on 28 days' test (N/mm^2)

No of Expt	Replicates	Response (N/mm^2)	Response Symbol	$\sum_{i=1}^{n} y_i$	$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$
1	A	18.4	Y_1	37.6	18.8
	B	19.2			
2	A	14.4	Y_2	29.8	14.9
	B	15.4			
3	A	14.5	Y_3	27.5	13.75
	B	13.0			
4	A	18.5	Y_4	39.0	19.5
	B	20.5			
5	A	15.0	Y_{12}	30.0	15
	B	15.0			
6	A	17.0	Y_{13}	35.0	17.5
	B	18.0			
7	A	15.0	Y_{14}	30.5	15.25
	B	15.0			
8	A	17.5	Y_{23}	33.5	16.75
	B	16.0			
9	A	13.5	Y_{24}	26.5	13.25
	B	13.0			

10	A	17.0	Y_{34}	33.0	16.5
	B	16.0			
11	A	15.5	C_1	29	14.5
	B	13.5			
12	A	18.5	C_2	35.5	17.75
	B	17.0			
13	A	15.0	C_3	30.5	15.25
	B	15.5			
14	A	16.0	C_4	33.0	16.5
	B	17.0			
15	A	14.0	C_5	27.5	13.75
	B	13.5			
16	A	16.5	C_5	31.0	15.50
	B	15.50			

4.1. Test of adequacy of the model

The formulated model was tested for adequacy using F-statistics (Table 5) at 5% level of significance. The tabulated F value was greater than the calculated F-value showing good reliability of the model. The results obtained from the computer programs coded in BASIC language are shown in the appendix. The computer program selects the appropriate mix ratios corresponding to any desired value of compressive strength accurately and without waste of time.

Table 5: F-statistics test of adequacy of the model

Control point	y_o	y_p	$y_o - \bar{y}_o$	$y_p - \bar{y}_p$	$(y_o - \bar{y}_o)^2$	$(y_p - \bar{y}_p)^2$
C1	14.5	12.630	-1.042	-2.073	1.085	4.299
C2	17.75	16.450	2.208	1.747	4.877	3.051
C3	15.25	11.960	-0.292	-2.743	0.085	7.526
C4	16.5	17.160	0.958	2.457	0.918	6.035
C5	13.75	13.330	-1.792	-1.373	3.210	1.886
C6	15.5	16.690	-0.042	1.987	0.002	3.947
	Σ =15.542	Σ =14.703			Σ =10.177	Σ =26.744

y_o , y_p = experimental and predicted compressive strength of periwinkle shell aggregate concrete respectively.

$\bar{y}_o = \frac{\Sigma y_o}{n}$; $\bar{y}_p = \frac{\Sigma y_p}{n}$

The sample variances S_o^2 and S_p^2 are determined as follows:

$$S_o{}^2 = \frac{10.177}{6} = 1.6962 \text{ ,}$$

$$S_p{}^2 = \frac{26.774}{6} = 4.4623$$

F is calculated as the ratio of the two sample variances as follows:

$$F = \frac{4.4623}{1.6962} = 2.631$$

From Fisher Table,

$$F_{0.95}(5,5) = 5.05$$

The table value of F (5.05) obtained from standard statistical table at 5% level of significance is greater than the calculated value of F (2.631) showing that the model is adequate.

5. Conclusions

The results of compressive strength obtained from the formulated model coincided with those obtained from experiment. The results obtained from F-statistics showed that the model is adequate. With the computer program written in BASIC language, the model predicts the mix ratios for any desired compressive strength value accurately and without waste of time.

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