

A Mathematical Model for the Prediction of Water Absorption of Chikoko-Cement Concrete

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Abstract

In this paper, a mathematical model is developed to predict the water absorption of chikoko-cement concrete mixes using Scheffe's regression function. A computer program, coded in BASIC language was used for easy prediction of water absorption of chikoko-cement concrete mixes. The results obtained from the mathematical model coincided with the experimental values at all points of observation. The model was tested for adequacy against the control values at 5% level of significance using F-statistics and was found to be adequate.

Keywords: *Mathematical model, water absorption, chikoko-cement concrete, regression function, computer program, level of significance, F-statistic*

1. Introduction

Concrete is the most widely used structural material in the world (Neville, 2003). The durability of concrete is largely dependent on the mix proportions of the component materials (Onwuka et al., 2013; Anyaogu and Ezeh, 2013; Obam and Osadebe, 2006). Researchers have shown that chikoko has been used as a natural pozzolana in concrete production, most especially in the Niger delta area of Nigeria in order to reduce the high cost of cement which is the most important component of concrete (Otoko, 2014; Onwuka and Sule, 2016; Onwuka and Sule, 2017). The alarming rate of structural failure and building collapse in Nigeria is not unconnected to improper and poor selection of proportions of concrete mixture components. The durability of concrete produced from partial replacement of cement with chikoko is of

paramount importance to engineers. Water absorption is an indicator of concrete durability (Neville, 2003). It is a long-term property of concrete that measures the ability of concrete to absorb and retain water (Shetty, 2005). The water absorption of hardened concrete is dependent on the proportions of the concrete mixture components. Therefore, proper selection of proportions of concrete mixture components remains a task of paramount importance to engineers, and this can be achieved by formulating a mathematical model in order to predict the mix proportions that will yield the required value of concrete water absorption at minimum cost (Okere et al., 2013).

The chikoko-cement concrete production in the Niger delta area of Nigeria is mainly by laborious laboratory trial mixes. The durability of concrete structures and houses produced from such mix proportions is not guaranteed as there

are no existing mathematical models and computer programs for easy prediction of water absorption and proper selection of the components mix proportions. In this paper, a mathematical model is developed to predict the water absorption and proportions of the component materials of chikoko-cement concrete mixture using Scheffe's regression function. A computer program coded in basic language is also developed for easy prediction of water absorption and proper selection of the mix ratios corresponding to the desired value of water absorption. The computer program is interactive.

2. Water absorption prediction model development

According to Scheffe (1958), the response function for a 5-component concrete mixture based on a (5, 2) factor space is given by:

$$\begin{aligned}
 Y = & \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + \alpha_5 X_5 + \alpha_{12} X_1 X_2 + \alpha_{13} X_1 X_3 \\
 & + \alpha_{14} X_1 X_4 + \alpha_{15} X_1 X_5 + \alpha_{23} X_2 X_3 + \alpha_{24} X_2 X_4 + \alpha_{25} X_2 X_5 + \alpha_{34} X_3 X_4 \\
 & + \alpha_{35} X_3 X_5 + \alpha_{45} X_4 X_5 \quad (1)
 \end{aligned}$$

Equation (1) is a bounded optimization function that is subject to the constraint:

$$\sum_{i=1}^5 X_i = 1 \quad (2)$$

where:

$X_i \geq 0$ are nonnegative concrete component proportions

$q =$ number of concrete mixture components

The coefficients of Equation (1) are obtained using Equations (3) and (4):

$$\alpha_i = Y_i \quad (3)$$

$$\alpha_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad (4)$$

where:

$$\alpha_i = \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_5$$

$$\alpha_{ij} = \alpha_{12}, \alpha_{13}, \alpha_{14}, \dots, \alpha_{45}$$

According to Scheffe, the required number of experimental points is a function of the number of mixture components and the degree of the chosen polynomial and it is given by:

$$N(q, m) = \frac{(q+m-1)!}{m!(q-1)!} \quad (5)$$

where:

$m, q =$ degree of the chosen polynomial

$q =$ number of concrete mixture components

In this study, $q = 5$ and $m = 2$. Substitution of these values into Equation (5), gives the number of experimental points as 15.

Let the actual and pseudo components be represented by Z_i and X_i respectively. The relationship between Z_i and X_i is given by:

$$X = BZ \quad (6)$$

where:

B is the inverse of S matrix

Similarly, according to Obam and Osadebe (2006), the inverse transformation from pseudo component, X_i to actual components, Z_i is given by:

$$Z = AX \quad (7)$$

where:

$A =$ the transpose of the actual mix ratios

$Z =$ actual mix ratio

$X =$ pseudo mix ratio

Using Equation (7), the actual mix ratios, Z_i are obtained and the transpose is given in matrix form as follows:

$$A = \begin{pmatrix} 0.526 & 0.566 & 0.589 & 0.611 & 0.596 \\ 0.947 & 0.919 & 0.823 & 0.889 & 0.846 \\ 0.053 & 0.081 & 0.177 & 0.111 & 0.154 \\ 2.100 & 2.020 & 0.191 & 2.160 & 2.150 \\ 4.200 & 4.040 & 3.820 & 4.320 & 4.300 \end{pmatrix} \quad (8)$$

2. Materials and methods

The cement used as binder was Dangote cement with properties meeting the requirements of BS 12:1978. The water used was fresh and free from organic matters. The fine aggregate was sourced from Otamiri River in Owerri Local Government Area of Imo State. The grading was carried out to the requirements of BS 812: Part 1, 1975. The sand belongs to grading zone 2. The granite used as coarse aggregate was obtained from a Crushed Rock Industry in Port Harcourt, Rivers State. The aggregate was thoroughly washed and sundried for two weeks to remove dirt. The chikoko was obtained from chikoko deposit at Eagle’s Island, Port Harcourt, Rivers State. It was sundried for two weeks after which it was ground and sieved to obtain fineness close to that of cement. The chemical

analysis was performed on chikoko sample to ascertain its suitability for use as a pozzolana in concrete production (Table 1). The design matrix for both the trial and control mixes obtained based on Scheffe’s (5, 2) factor spaces are shown on Tables 2 and 3 respectively. The additional fifteen runs of experiment were used as check points for model validation.

3.1. Water absorption test

Water absorption test was carried out according to the recommendations of BS 1881-122 (983). Water absorption test was performed on chikoko-cement concrete cubes after 28 days of immersion in water. The samples were removed from the curing tank and then sundried until there was no change in mass. The concrete cubes were weighed and recorded as dry masses. The samples were then immersed in water for 24 hours and their wet masses were recorded. The difference in mass represents the mass of water absorbed by the concrete cube. The water absorption was calculated using the formula:

$$W_a = \frac{M_w - M_d}{M_d} * 100\% \quad (9)$$

where W_a = water absorption, M_w = mass of wet cube, M_d = mass of dry cube

Table 1: Chemical properties of chikoko

S/No	Component	Content (%)
1	CaO	9.85
2	SiO ₂	41.21
3	Al ₂ O ₃	10.15
4	Fe ₂ O ₃	2.31
5	MgO	5.02
6	Na ₂ O	1.97
7	K ₂ O	8.17
8	SO ₃	0.08
9	TiO ₂	0.72

10	ZnO	0.09
11	LoI	6.51

Table 2: Design matrix for trial points based on Scheffe’s (5, 2) factor space

Z_1	Z_2	Z_3	Z_4	Z_5	Response	X_1	X_2	X_3	X_4	X_5
0.52601	0.947	0.053	2.1	4.2	Y ₁	1	0	0	0	0
0.566	0.91901	0.081	2.02	4.04	Y ₂	0	1	0	0	0
0.589	0.823	0.17701	1.91	3.82	Y ₃	0	0	1	0	0
0.611	0.889	0.111	2.1601	4.32	Y ₄	0	0	0	1	0
0.596	0.846	0.154	2.15	4.301	Y ₅	0	0	0	0	1
0.546005	0.933005	0.067	2.06	4.12	Y ₁₂	0.50	0.50	0	0	0
0.557505	0.885	0.115005	2.005	4.01	Y ₁₃	0.50	0	0.50	0	0
0.568505	0.918	0.082	2.13005	4.26	Y ₁₄	0.50	0	0	0.50	0
0.561005	0.8965	0.1035	2.125	4.2505	Y ₁₅	0.50	0	0	0	0.50
0.5775	0.871005	0.129005	1.965	3.93	Y ₂₃	0	0.50	0.50	0	0
0.5885	0.904005	0.096	2.09005	4.18	Y ₂₄	0	0.50	0	0.50	0
0.581	0.882505	0.1175	2.085	4.1705	Y ₂₅	0	0.50	0	0	0.50
0.6	0.856	0.144005	2.03505	4.07	Y ₃₄	0	0	0.50	0.50	0
0.5925	0.8345	0.165505	2.03	4.0605	Y ₃₅	0	0	0.50	0	0.50
0.6035	0.8675	0.1325	2.15505	4.3105	Y ₄₅	0	0	0	0.50	0.50

Table 3: Design matrix for control points based on Scheffe’s (5, 2) factor space

Z_1	Z_2	Z_3	Z_4	Z_5	Response	X_1	X_2	X_3	X_4	X_5
0.560337	0.896337	0.10367	2.01	4.02	C ₁	0.333333	0.333333	0.333333	0	0
0.575337	0.886333	0.11367	2.0567	4.11333333	C ₂	0.333333	0	0.333333	0.333333	0
0.57767	0.894	0.106	2.1367	4.27366666	C ₃	0.333333	0	0	0.333333	0.333333
0.573003	0.894503	0.105503	2.047525	4.095	C ₄	0.25	0.25	0.25	0.25	0
0.580503	0.87625	0.123753	2.080025	4.16025	C ₅	0.25	0	0.25	0.25	0.25
0.569253	0.883753	0.116253	2.045	4.09025	C ₁₂	0.25	0.25	0.25	0	0.25
0.551755	0.909003	0.091003	2.0325	4.065	C ₁₃	0.5	0.25	0.25	0	0
0.576753	0.8655	0.134503	2.0775	4.1555	C ₁₄	0.25	0	0.25	0	0.5
0.563604	0.905002	0.095002	2.05802	4.116	C ₁₅	0.4	0.2	0.2	0.2	0
0.577602	0.884802	0.115202	2.06802	4.1362	C ₂₃	0.2	0.2	0.2	0.2	0.2
0.573603	0.887601	0.112402	2.07602	4.1522	C ₂₄	0.3	0.1	0.2	0.2	0.2
0.586101	0.879002	0.121002	2.07403	4.1482	C ₂₅	0.1	0.2	0.2	0.3	0.2
0.565254	0.886552	0.113453	2.053	4.10625	C ₃₄	0.35	0.15	0.25	0	0.25
0.574453	0.891002	0.109002	2.07752	4.1552	C ₃₅	0.25	0.2	0.15	0.2	0.2
0.566005	0.903701	0.0963	2.12302	4.2463	C ₄₅	0.45	0.05	0	0.2	0.3

where:

Z_1 = actual proportion of water

X_1 = pseudo proportion of water

Z_2 = actual proportion of cement

X_2 = pseudo proportion of cement

Z_3 = actual proportion of chikoko X_3 = pseudo proportion of chikoko
 Z_4 = actual proportion of sand X_4 = pseudo proportion of sand
 Z_5 = actual proportion of granite X_5 = pseudo proportion of granite

4. Results and discussion

The results of water absorption test after 28 days curing in water are as given in Table 4. From Equations (3) and (4) and Table 4, the coefficients of Equation (1) are obtained as follows:

$$\alpha_1 = 0.922, \alpha_2 = 0.839, \alpha_3 = 1.221, \alpha_4 = 1.124, \alpha_5 = 1.135, \alpha_{12} = 0.306, \alpha_{13} = 2.146$$

$$\alpha_{14} = -0.736, \alpha_{15} = -0.594, \alpha_{23} = -0.904, \alpha_{24} = -0.402, \alpha_{25} = 0.556, \alpha_{34} = -0.142$$

$$\alpha_{35} = -0.648, \alpha_{45} = -1.13$$

Substitution of the obtained coefficients, α_i into Equation (1) yields:

$$y = 0.922X_1 + 0.839X_2 + 1.221X_3 + 1.124X_4 + 1.135X_5 + 0.306X_1X_2 + 2.146X_1X_3 - 0.736X_1X_4 - 0.594X_1X_5 - 0.904X_2X_3 - 0.402X_2X_4 + 0.556X_2X_5 - 0.142X_3X_4 - 0.648X_3X_5 - 1.13X_4X_5 \tag{10}$$

Equation (10) is the mathematical model model for the prediction of 28th day water absorption of chikoko-cement concrete based on Scheffe’s (5, 2) factor space.

Table 4: Water absorption test results of chikoko-cement concrete

No of Observations	Replicates	Dry Mass (Kg)	Wet Mass (Kg)	Water Absorption (%)	Average water Absorption (%)
1	A	8.34	8.41	0.839	
	B	8.30	8.38	0.964	0.922
	C	8.30	8.38	0.964	
2	A	8.35	8.42	0.838	
	B	8.35	8.42	0.838	0.839
	C	8.33	8.40	0.840	
3	A	8.22	8.31	1.095	
	B	8.13	8.25	1.476	1.221
	C	8.25	8.34	1.091	
4	A	8.30	8.38	0.964	
	B	8.31	8.42	1.324	1.124
	C	8.30	8.39	1.084	
5	A	8.23	8.33	1.215	
	B	8.21	8.31	1.218	1.135
	C	8.22	8.30	0.973	
6	A	8.38	8.46	0.955	
	B	8.35	8.43	0.958	0.957
	C	8.35	8.43	0.958	
7	A	8.31	8.45	1.685	
	B	8.29	8.44	1.809	1.608

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	C	8.27	8.38	1.330	
8	A	8.36	8.43	0.837	
	B	8.34	8.42	0.959	0.839
	C	8.34	8.40	0.719	
9	A	8.36	8.43	0.837	
	B	8.33	8.41	0.960	0.880
	C	8.31	8.38	0.842	
10	A	8.27	8.34	0.846	
	B	8.28	8.34	0.725	0.804
	C	8.31	8.38	0.842	
11	A	8.33	8.41	0.960	
	B	8.32	8.39	0.841	0.881
	C	8.33	8.40	0.840	
12	A	8.28	8.37	1.087	
	B	8.27	8.36	1.088	1.126
	C	8.32	8.42	1.202	
13	A	8.23	8.32	1.094	
	B	8.20	8.29	1.098	1.137
	C	8.20	8.30	1.220	
14	A	8.21	8.30	1.096	
	B	8.21	8.29	0.974	1.016
	C	8.18	8.26	0.978	
15	A	8.27	8.34	0.846	
	B	8.26	8.33	0.847	0.847
	C	8.25	8.32	0.848	
Check points for model validation					
C1	A	8.32	8.44	1.442	
	B	8.32	8.43	1.322	1.283
	C	8.30	8.39	1.084	
C2	A	8.31	8.39	0.963	
	B	8.31	8.39	0.963	0.964
	C	8.28	8.36	0.966	
C3	A	8.34	8.41	0.839	
	B	8.34	8.40	0.719	0.800
	C	8.31	8.38	0.842	
C4	A	8.31	8.37	0.722	
	B	8.30	8.38	0.964	0.922
	C	8.33	8.42	1.080	
C5	A	8.27	8.35	0.967	
	B	8.27	8.36	1.088	1.046
	C	8.31	8.40	1.083	
C6	A	8.27	8.35	0.967	
	B	8.32	8.42	1.202	1.045
	C	8.28	8.36	0.966	

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C7	A	8.34	8.42	0.959	
	B	8.34	8.44	1.199	1.039
	C	8.34	8.42	0.959	
C8	A	8.27	8.38	1.330	
	B	8.26	8.34	0.969	1.049
	C	8.26	8.33	0.847	
C9	A	8.32	8.40	0.962	
	B	8.33	8.41	0.960	1.001
	C	8.33	8.42	1.080	
C10	A	8.30	8.39	1.084	
	B	8.31	8.39	0.963	1.045
	C	8.28	8.37	1.087	
C11	A	8.30	8.39	1.084	
	B	8.31	8.38	0.842	1.003
	C	8.31	8.40	1.083	
C12	A	8.28	8.37	1.087	
	B	8.28	8.36	0.966	1.045
	C	8.31	8.40	1.083	
C13	A	8.28	8.35	0.845	
	B	8.30	8.36	0.723	1.006
	C	8.28	8.40	1.449	
C14	A	8.29	8.37	0.965	
	B	8.32	8.41	1.082	0.963
	C	8.32	8.39	0.841	
C15	A	8.34	8.41	0.839	
	B	8.32	8.39	0.841	0.840
	C	8.34	8.41	0.839	

4.1. Test of adequacy of the model

The model was tested for goodness of fit against the check points using F-statistic at 5% level of significance. The results are as shown on Table 5.

Table 5: F-statistic test for water absorption test results

Control points	y_o	y_p	$y_o - \bar{y}_o$	$y_p - \bar{y}_p$	$(y_o - \bar{y}_o)^2$	$(y_p - \bar{y}_p)^2$
C1	1.283	1.166	0.280	0.125	0.078	0.016
C2	0.964	1.230	-0.039	0.189	0.002	0.036
C3	0.8	0.787	-0.203	-0.254	0.041	0.064
C4	0.922	1.043	-0.081	0.002	0.007	0.000
C5	1.046	1.032	0.043	-0.009	0.002	0.000
C6	1.045	1.083	0.042	0.042	0.002	0.002
C7	1.039	1.226	0.036	0.185	0.001	0.034
C8	1.049	1.082	0.046	0.041	0.002	0.002
C9	1.001	1.085	-0.002	0.044	0.000	0.002
C10	1.045	0.986	0.042	-0.055	0.002	0.003

C11	1.003	1.023	0.000	-0.018	0.000	0.000
C12	1.045	0.943	0.042	-0.098	0.002	0.010
C13	1.006	1.136	0.003	0.095	0.000	0.009
C14	0.963	0.973	-0.040	-0.068	0.002	0.005
C15	0.840	0.819	-0.163	-0.222	0.027	0.05
Σ	1.003	1.041		Σ	0.166	0.231

where:

y_o, y_p = observed and predicted value of water absorption respectively

$$\bar{y}_o = \frac{\Sigma y_o}{n}; \quad \bar{y}_p = \frac{\Sigma y_p}{n}$$

\bar{y}_o, \bar{y}_p = mean of observed and predicted value of water absorption respectively

$$S_o^2 = \frac{0.166}{14} = 0.0118874, \quad S_p^2 = \frac{0.231}{14} = 0.01652292$$

S_o^2, S_p^2 = variance of observed and predicted value of water absorption respectively

$$F = \frac{S_1^2}{S_2^2} \quad (11)$$

where:

S_1^2, S_2^2 = larger and smaller value of sample variance respectively

$$F = \frac{0.01652292}{0.0118874} = 1.38995$$

From statistical Table, $F_{0.95}(14,14) = 2.4$

The calculated value of F is less than the table value. The model is therefore adequate for the prediction of water absorption of chikoko-cement concrete.

4. Conclusions

A mathematical model has been developed to predict the water absorption of chikoko-cement concrete and has been tested for adequacy against the check points using F-statistic. The results obtained from F-statistic (Table 5) showed that the regression model is adequate for the prediction of water absorption of chikoko-cement concrete. The predicted and experimental values of water absorption are almost identical at all points of observation. The water absorption of chikoko-cement concrete depends on the proportions of the concrete component materials: water, cement, chikoko, sand and coarse aggregates. A computer program developed based on the mathematical model can be used to select the mix ratios of chikoko blended cement

concrete corresponding to a desired value of water absorption and vice-versa.

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