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Reliability Analysis of an I-Section Steel Beam

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Abstract
In this paper, a reliability analysis of an I-Section Steel beam is carried out based on Advanced First Order Second Moment format. The limit state function was developed considering shear failure mode of the I-Steel beam. The random variables used for the reliability estimation were obtained from literature. A MATLAB script was developed based on the shear limit state function developed and a MATLAB routine solver, fmincon, was invoked in the reliability estimation. The reliability index (4.7941) obtained based on the present method was validated using the result obtained using a solver routine in Microsoft Excel and the two results were found to be identical. The reliability index obtained based on the new approach was also compared with the value obtained from literature (4.796) and the two values were also found to almost identical.

***Key words:** Reliability analysis, limit state function, fmincon, random variables, failure mode*

1. Introduction

Civil engineering structures must satisfy both the ultimate and serviceability requirements in service (Mosley and Bungey, 1990). Civil engineering structures deteriorate over time and this is a major reason for structural reliability analysis (Melchers, 1999; Sule, 2011; Benu and Sule, 2012; Sule and Eme, 2014; Sule, 2013). The structural design parameters used in the conventional civil engineering design are uncertain. Consequently, it is difficult to guarantee the safety of structures through the partial safety factors used in the conventional design equations (Abubakar, 1996). The recent increase in fatality rate and damage of properties resulting from structural failure and eventual collapse of structures is a source of concern to both civil and structural engineers. According to Afolayan (2004), reliability analysis of a civil engineering structure is a necessity at every stage of the service live of structures rather than sitting down and watch the structures collapse. Some distressed structures in Nigeria have collapsed resulting in loss of many lives and damage of properties worth millions of naira (Obot and Archibong, 2016). Concrete structures fail without warning and the failure of a flexural beam in shear is associated with multiple cracks. Consequently, the timely intervention of the structural engineer is required in order to carry out a reliability analysis on the affected structure or structural component in order to forestall the damages it might cause when it eventually collapses.

In this paper, a limit state function was developed for an I-Section Steel beam based on shear failure mode using design parameters obtained from literature. A MATLAB program was developed based on the limit state function to estimate the reliability index of the beam. The results obtained from the MATLAB code were validated using the results obtained from a solver routine in Microsoft Excel and the two results were found to be identical. Also, the results obtained based on the present method were compared with the results obtained from literature and were found to agree favourably.

2. Structural reliability model derivation

The performance function is the random difference between the resistance of a structure $R(X)$ and the load effects $S(X)$. For n-dimensional random variables the performance function is given by:

$$g(X) = g(X_1, X_2, X_3, \ldots\ldots\ldots, X_n) = R(X) - S(X) \quad (1)$$

where: $X_1, X_2, X_3, \ldots\ldots\ldots, X_n$ are the basic design variables; $R(X)$ and $S(X)$ is structural resistance and load effect

From Equation (1), the limit state equation is given by:

$$g(X) = g(X_1, X_2, X_3, \ldots\ldots\ldots, X_n) = 0 \quad (2)$$

According to Melchers (1999), $g(X) = 0$ is the limit state surface and it represents the boundary between the safe and unsafe domain of the structure, $g(X) > 0$ represents the safe condition of a structure while $g(X) < 0$ represents the unsafe condition of a structure.

Non-normal random variables X_i are transformed into independent normal variables using the transformation:

$$Z^* = \frac{X_i - \mu_{xi}}{\sigma_{xi}} \, , \quad (i = 1, 2, \ldots.., n) \quad (3)$$

Using Equation (3), the limit state equation for n transformed variates is given by:

$$g\begin{pmatrix} \sigma^{x1} * Z^{1^*} + \mu^{x1}, \; \sigma^{x2} * Z^{2^*} + \mu^{x2}, \\ \sigma^{x3} * Z^{3^*} + \mu^{x3}, \ldots\ldots\ldots, \sigma^{xn} * Z^{n^*} + \mu^{xn} \end{pmatrix} = 0 \quad (4)$$

The vector of the design point on the failure surface $g(X_i^*) = 0$ to the origin of the design variable space is given by:

$$Z^* = \left(Z_1^*, \; Z_2^*, \; Z_3^*, \; Z_n^* \right) \quad (5)$$

The structural reliability index β is the minimum from the most probable failure point on the failure surface $g(X_i^*) = 0$ to the origin of the design variable space. This minimum distance is given by:

$$\|D\| = \beta = \sqrt{\left(Z_1^{*2} + Z_2^{*2} + Z_3^{*2} + \ldots\ldots + Z_n^{*2}\right)} \quad (6)$$

Equation (6) has a global minimum at the design point $Z^* = \left(Z_1^*, \; Z_2^*, \; Z_3^*, \; Z_n^* \right)$ on the failure surface, $g(X_i^*) = 0$. The design point $Z^* = \left(Z_1^*, \; Z_2^*, \; Z_3^*, \; Z_n^* \right)$ is determined by minimizing the equation (6) subject to:

$$g(X_i) = 0 \quad (7)$$

The design points $Z^* = \left(Z_1^*, \; Z_2^*, \; Z_3^*, \; Z_n^* \right)$ are the values of Z which satisfy equation (7). According to Ranganathan (1990), the expression that relates the design point, Z_i^* in transformed variables on the failure surface, the reliability index, β and the direction cosines, α_i is given by:

$$Z_i^* = \beta * \alpha_i \quad (8)$$

where: α_i = direction cosines along the coordinate axes Z_i^*

3. Applicability of method

An example problem (Figure 1) obtained from literature (Ranganathan, 1990), is used to demonstrate the applicability of the present approach to structural reliability estimation.

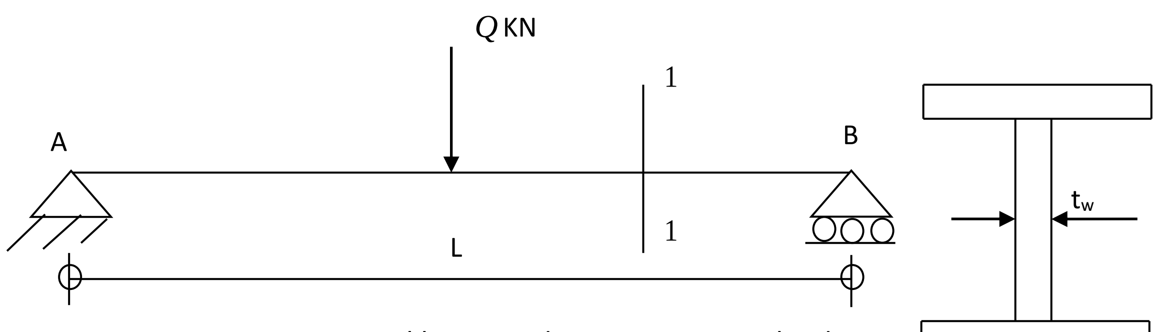

Figure 1: An I Section steel beam under a centre point loading

Section 1 - 1

The basic variable descriptive statistics (Ranganatan, 1990) are:

$\mu_Q = 4000N;\ \sigma_Q = 1000N$

$\mu_{fS} = 95\,N/mm^2;\ \sigma_{fS} = 10\,N/mm^2$

$\sigma_d = 2.5mm;\ \frac{d}{t_w} = 40;\ \mu_d = 50mm$

where: d = depth of the beam, t_w = web thickness and f_S = shear strength of the material t_w is treated as a deterministic parameter. The probability distribution of all the basic variables is assumed to be normal.

From Figure1, the maximum shear force representing the load effect at the support is given by:

$$S(Q) = \frac{Q}{2} \quad (9)$$

The resistance offered by the web in shear is given by:

$$R(f_s, t_w, d) = f_S * t_w * d \quad (10)$$

The violation of limit state in shear is given by:

$$f_S * t_w * d - \frac{Q}{2} \leq 0 \quad (11)$$

Using Equations (10) and (11), the limit state equation becomes:

$$g(X) = f_S * t_w * d - \frac{Q}{2} = 0 \quad (12)$$

The basic mean and standard deviation o the basic variables are substituted in Equation (4)
in equation (4) and after simplifying yields:

$$g(Z) = 625 * Z_1 + 296.88 * Z_2 + 31.25 * Z_1 * Z_2 - 500 * Z_3 + 3937.5 = 0 \quad (13)$$

According to Ranganathan (1990), the expression that relates the design point, Z_i^* in transformed variables on the failure surface, the reliability index, β and the direction cosines α_i is given by:

$$Z_i^* = \beta * \alpha_i \quad (14)$$

where: α_i = direction cosines along the coordinate axes Z_i^*

4. Results and discussion

Figure 2 is the Microsoft Excel Solver dialogue box while Table 1 is the spreadsheet for reliability analysis of an I-Section Steel beam at the limit state of shear used in this study. The advanced Second Moment format has been employed for the reliability analysis. The reliability index and direction cosines obtained from the present approach are found to be in consonance with those obtained using a solver routine in Microsoft Excel (Reliability index = 4.7941, alpha 1 = -0.742, alpha 2 = -0.234 and alpha 3 = 0.629). These values are found to agree favourably with the results (Reliability index = 4.796, alpha 1 = -0.741, alpha 2 = -0.234 and alpha 3 = 0.629) obtained from literature (Ranganathan, 1990), showing good reliability of the present approach.

% A MATLAB Script that minimizes the Euclidean distance, Beta

```
Function Beta = objfun2(x)
Beta = sqrt(x(1)^2+x(2)^2+x(3)^2)
NatafTransform = 0;
end
% Constraint function
Function [c,ceq] = confun2(x)
C = [625*x(1)+296.88*x(2)+31.25*x(1)*x(2)-500*x(3)+3937.5];
% nonlinear inequality constraint
ceq = [ ];
end
% Invoke constrained optimization routine
X0 = [1,1,1]; % Initial guess values of basic design variables
Options = optimset('LargeScale','off');
[X, fval] = fmincon('objfun2',X0,[],[],[],[],[],[],'confun2',options)
Program outputs:
Local minimum found that satisfies the constraints
x = -3.556 -1.119   3.014
fval = 4.7941
```

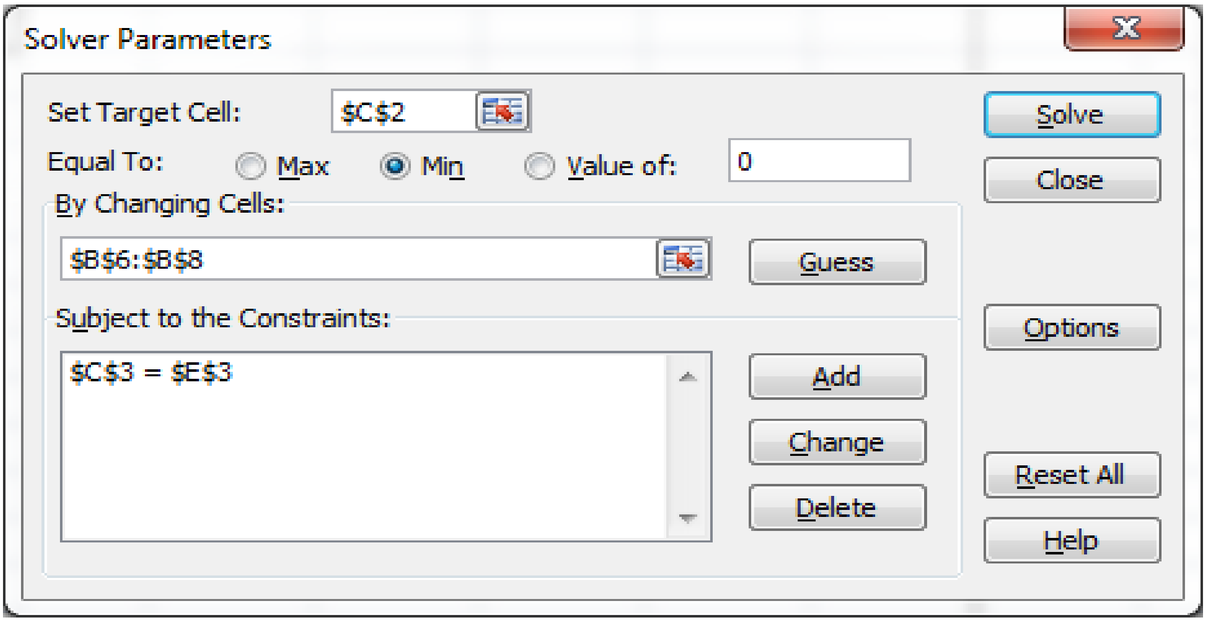

Figure 2: A Microsoft Excel Solver dialogue box

Table 1: Spreadsheet for reliability analysis of an I-Section Steel beam at the limit state of shear

	A	B	C	D	E	F	G	H	I
1									
2	**Objective function (Beta)=**		4.7941						
3	**Design constraint g(X) =**		-4E-12	Equal	0				
4									
5	**Design points on failure surface**					**Optimization models**			
6	Z1 =	-3.556		Determi	Beta =	SQRT(B6*B6+B7*B7+B8*B8)			S. t.
7	Z2 =	-1.120		g(X) =	625*B6+296.88*B7+31.25*B6*B7-500*B8+3937.5				
8	Z3 =	3.014							
9									
10		**Direction cosines**							
11		Alpha 1=	-0.742						
12		Alpha 2=	-0.234						
13		Alpha 3=	0.629						
14									
15									

5. Conclusions

The advanced Second Moment format has been employed for the reliability analysis of an I-Steel beam at the limit state of shear. The reliability index and direction cosines obtained based on the present approach are found to be in consonance with those obtained using a solver routine in Microsoft Excel and are also identical with those obtained from literature showing that the present approach can be used as a tool for reliability analysis of structural members and structures.

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