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Errors and Stability Analysis of a New Numerical Method of Solution for Heat Conduction Equations

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Abstract
In this work, a new numerical finite difference scheme for the solution of heat conduction equation arising from heat conduction was developed due to the recent growing interest in the derivation of continuous numerical finite difference method for solving Partial Differential Equations (PDE). This was done based on the collocation and interpolation of the PDE directly over multi-steps along lines but without reduction to a system of Ordinary Differential Equations (ODE). The intention was to avoid the cost of solving a large system of coupled ODEs often arising from the reduction method by a semi-discretization. The performance of the new numerical finite difference method was tested. The numerical results obtained showed that the method provided better results than the known explicit finite difference method. There was no semi-discretization involved in the derivation of this scheme, and no reduction of PDE to a system of ODE was recorded, but rather a system of algebraic equations was formulated. Therefore, the desire was to derive a new scheme that will be used in finding the solutions of the system of algebraic equations formulated from the discretization of the heat conduction equations with respect to the space and time variables. This new numerical method was applied to solve two different test problems with known solutions. Also, the error and stability analysis of the new scheme was investigated. Detailed results of these have shown that the new scheme was more stable with lesser error than the known Schmidt explicit method.

Keywords: *Lines, Multistep collocation, Parabolic, Taylor's polynomial*

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1. Introduction

There are some salient problems associated with the derivation of the known explicit method for solving PDE arising from heat conduction equations. The earlier method used in the derivation of Schmidt method seeks for the functional evaluation of some complex functions of heat diffusion equations, and also employs the Taylor series expansion. These functional evaluations and expansions might have introduced errors due to truncation (Biazar and Ebrahimi, 2005; Chawla and Katti, 1979; Yakubu et al., 2004). Based on that, there is of recent a growing interest in literature to seek an alternative method of its derivation. For an efficient algorithm, therefore, there is the need to try to eliminate these problems. And algorithms that can overcome these problems will be of advantage (Crandall, 1995; Crane and Klopfenstein, 1965).

Following Awoyemi (2002), Crank (1947) and Onumanyi et al. (2002), a single PDE in one space variable, where t and x are the time and space coordinates respectively, and the quantities h and k are the mesh sizes in the space and time directions is considered. The interest is to extend the continuous numerical work of Sirisena et al. (2001) and Dehghan (2003) to obtain another new continuous numerical method that can solve the PDE arising from heat conduction equations. This is done based on the collocation and interpolation of the PDE directly over multi steps along lines but without reduction to a system of ODEs (Bao et al., 2003). The derivation avoids the cost of solving a large system of coupled ODEs often arising from the reduction method by a semi - discretization. The new method also helps eliminates the usual draw-back of stiffness arising in the conventional reduction method by semi-discretization as suggested by Awoyemi (1998) and Dieci (1992).

The new scheme is applied to solve two different test problems with known exact solutions. The results obtained are compared with the results from Schmidt explicit method, and with the exact solutions obtained from analytic method. From the error and stability analysis, one could see that the error in the new scheme is minimal, and in terms of stability the new scheme with $r \leq \frac{1}{30}$ is more stable than the Schmidt explicit scheme. The numerical results confirmed the efficacy and the validity of the new numerical scheme and suggested that it is an interesting and viable numerical method.

2. The solution method

For such a new continuous numerical method to be developed, Equation (1) with its associated initial and boundary conditions was used (Odekunle, 2003).

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \quad 0 \leq x \leq b, \; 0 \leq t \leq T \tag{1}$$

Subject to the initial and boundary conditions

$$U(x,0) = f(x), \; 0 \leq x \leq b,$$

$$U(0,t) = g_1(t), \; t \geq 0$$
$$U(b,t) = g_2(t), \; t \geq 0$$

The interval $0 \leq x \leq b$ is subdivided into N equal subintervals by the grid points $x_m = mh, \quad m = 0,\ldots, N$ where $Nh = b$. On these meshes we seek $l-$step approximate solution to $U(x,t)$ of the form

$$U(x,t) = \sum_{r=0}^{p-2} a_r Q_r(x,t) \qquad x \in [x_m, x_{m+l}] \tag{2}$$

such that $0 = x_0 < \ldots < x_m < \ldots < x_N = b$ (Brown, 1979). The basis function $Q_r(x,t), \; r = 0,\ldots, p-2$ in Equation (2) are assumed known, a_r are constants to be determined and $p \leq l + s$, where s is the number of collocation points. The equality holds if the number of interpolation points used is equal to l according to Sirisena and Onumanyi (1994) and Yakonov (1963). There will be flexibility in the choice of the basis function $Q_r(x,t)$ as may be desired for specific application. For this work, the Taylor's polynomial $Q_r(x,t) = x^r t^r$ is considered. The interpolation values $U_{m,n},\ldots,U_{m+l-1,n}$ are assumed to have been determined from previous steps, while the method seeks to obtain $U_{m+l,n}$ (Biazar and Ebrahimi, 2005; Weideman and Herbst, 1986). Applying the above interpolation conditions on Equation (2) gave Equation (3).

$$a_0 Q_0(x_{m+g}, t_n) + \ldots + a_{p-2} Q_{p-2}(x_{m+g}, t_n) = U(x_{m+g}, t_n), \quad g = -\frac{1}{15}\left(\frac{1}{15}\right) l - \frac{44}{15} \tag{3}$$

And by Yahaya and Onumanyi (1997), Equation. (3) can be written as a simple matrix equation in the augmented form as:

$$\begin{bmatrix} Q_0\left(x_{m-\frac{1}{15}}, t_n\right) & \cdots & Q_{p-2}(x_{m-\frac{1}{15}}, t_n) \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ Q_0\left(x_{m+l-\frac{44}{15}}, t_n\right) & \cdots & Q_{p-2}(x_{m+l-\frac{44}{15}}, t_n) \end{bmatrix} \begin{bmatrix} a_0 \\ \cdots \\ \cdots \\ \cdots \\ a_{p-2} \end{bmatrix} = \begin{bmatrix} U(x_{m-\frac{1}{15}}, t_n) \\ \cdots \\ \cdots \\ \cdots \\ U(x_{m+l-\frac{44}{15}}, t_n) \end{bmatrix} \tag{4}$$

If we use three interpolation points and one collocation point, this implies that $s = 1$, $p = 4$, $l = 3$ and $r = 0,1,2$. Substituting for p in Equation (2) gave Equation (5).

$$a_0 Q_0(x_{m+g}, t_n) + a_1 Q_1(x_{m+g}, t_n) + a_2 Q_2(x_{m+g}, t_n) = U_{m+g,n} \qquad g = -\frac{1}{15}, 0, \frac{1}{15} \tag{5}$$

Putting the values of g in Equation (4) and writing it as matrix in augmented form gave Equation (6).

$$\begin{bmatrix} Q_0\left(x_{m-\frac{1}{15}},t_n\right) & Q_1\left(x_{m-\frac{1}{15}},t_n\right) & Q_2(x_{m-\frac{1}{15}},t_n) \\ Q_0(x_m,t_n) & Q_1(x_m,t_n) & Q_2(x_m,t_n) \\ Q_0\left(x_{m+\frac{1}{15}},t_n\right) & Q_1\left(x_{m+\frac{1}{15}},t_n\right) & Q_2(x_{m+\frac{1}{15}},t_n) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_3 \end{bmatrix} = \begin{bmatrix} U(x_{m-\frac{1}{15}},t_n) \\ U(x_m,t_n) \\ U(x_{m+\frac{1}{15}},t_n) \end{bmatrix} \tag{6}$$

From Equation (6) the following values were obtained:

$$\left\{\begin{matrix} Q_0\left(x_{m-\frac{1}{15}},t_n\right)=1 & Q_1\left(x_{m-\frac{1}{15}},t_n\right)=x_{m-\frac{1}{15}}t_n & Q_2(x_{m-\frac{1}{15}},t_n)=x^2{}_{m-\frac{1}{15}}t^2{}_n \\ Q_0(x_m,t_n)=1 & Q_1(x_m,t_n)=x_m t_n & Q_2(x_m,t_n)=x^2{}_m t^2{}_n \\ Q_0\left(x_{m+\frac{1}{15}},t_n\right)=1 & Q_1\left(x_{m+\frac{1}{15}},t_n\right)=x_{m+\frac{1}{15}}t_n & Q_2(x_{m+\frac{1}{15}},t_n)=x^2{}_{m+\frac{1}{15}}t_n{}^2 \end{matrix}\right\} \tag{7}$$

Putting the values of Equation (7) into Equation (6) gave Equation (8).

$$\begin{bmatrix} 1 & x_{m-\frac{1}{15}}t_n & x^2{}_{m-\frac{1}{15}}t^2{}_n \\ 1 & x_m t_n & x^2{}_m t^2{}_n \\ 1 & x_{m+\frac{1}{15}}t_n & x^2{}_{m+\frac{1}{15}}t^2{}_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} U_{m-\frac{1}{15},n} \\ U_{m,n} \\ U_{m+\frac{1}{15},n} \end{bmatrix} \tag{8}$$

When Equation (8) is solved, a_2 is obtained as

$$a_2 = \frac{225\left(U_{m+\frac{1}{15},n} + U_{m-\frac{1}{15},n} - 2U_{m,n}\right)}{2h^2 t^2{}_n},$$

substituting $r = 0,1,2$ in Equation (1) gave Equation (9).

$$U(x,t) = a_0 Q_0 + a_1 Q_1 + a_2 Q_2 \tag{9}$$

The substitution of Q_0 Q_1 *and* Q_2 in Equation (9) gave Equation (10).

$$U(x,t) = a_0 + a_1 xt + a_2 x^2 t^2 \tag{10}$$

Substituting the value of a_2 in Equation (10) gave Equation (11).

$$U(x,t) = a_0 + a_1 xt + x^2 t^2 \left(\frac{225U_{m+\frac{1}{15},n} + 225U_{m-\frac{1}{15},n} - 450U_{m,n}}{2h^2 t^2{}_n} \right) \tag{11}$$

Taking the first and second derivatives of Equation (11) with respect to x yielded

$$U''(x,t) = t^2\left(\frac{225U_{m+\frac{1}{15},n} + 225U_{m-\frac{1}{15},n} - 450U_{m,n}}{h^2 t^2_n}\right), \tag{12}$$

After collocating Equation (12) at $t = t_n$ it became

$$U''(x,t) = \frac{225U_{m+\frac{1}{15},n} + 225U_{m-\frac{1}{15},n} - 450U_{m,n}}{h^2} \tag{13}$$

According to Odekunle et al. (2003) and Penzl (2000), roles of x and t in Equation (2) is reversed, and the interval $0 \le t \le T$ is subdivided into y equal subintervals by the grid points $t_n = nk, \quad n = 0,\ldots,y$ where $yk = T$. On these meshes the $l-$step approximate solution to $U(x,t)$ of the form

$$U(x,t) = \sum_{r=0}^{p-2} a_r Q_r(x,t) \qquad t \in [t_n, t_{n+l}] \tag{14}$$

such that $0 = t_0 < \ldots < t_n < \ldots < t_y = T$. The basis function $Q_r(x,t), \; r = 0,\ldots,p-2$ in Equation (14) are assumed to have been known, and a_r are constants to be determined and $p \le l + s$, where s is the number of collocation points. The equality holds if the number of interpolation points used is equal to l. There will be flexibility in the choice of the basis function $Q_r(x,t)$ as may be desired for specific application. For this method, the Taylor's polynomial $Q_r(x,t) = x^r t^r$ is considered. The interpolation values $U_{m,n},\ldots,U_{m,n+l-1}$ are assumed to have been determined from previous steps, while the method seeks to obtain $U_{m,n+l}$ (Pierre, 2008; Sirisena et al., 1996). Applying the interpolation conditions on Equation (14) produced.

$$a_0 Q_0(x_m, t_{n+f}) + \ldots + a_{p-2} Q_{p-2}(x_m, t_{n+f}) = U(x_m, t_{n+f}), \text{ we assume } f = 0\left(\frac{1}{15}\right) l - \frac{29}{15} \tag{15}$$

Equation (15) can be written as a simple matrix equation in the augmented form as:

$$\begin{bmatrix} Q_0(x_m,t_n) & \ldots & Q_{p-2}(x_m,t_n) \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ Q_0(x_m,t_{n+l-\frac{29}{15}}) & \ldots & Q_{p-2}(x_m,t_{n+l-\frac{29}{15}}) \end{bmatrix} \begin{bmatrix} a_0 \\ \ldots \\ \ldots \\ \ldots \\ a_{p-2} \end{bmatrix} = \begin{bmatrix} U(x_m,t_n) \\ \ldots \\ \ldots \\ \ldots \\ U(x_m,t_{n+l-\frac{29}{15}}) \end{bmatrix} \tag{16}$$

Using two interpolation points and one collocation point in Equation (16) implies that $p = 3, r = 0,1 l = 2$ and $f = 0, \frac{1}{15}$, and by substitution Equation (16) becomes:

$$\begin{bmatrix} Q_0(x_m,t_n) & Q_1(x_m,t_n) \\ Q_0\left(x_m,t_{n+\frac{1}{15}}\right) & Q_1\left(x_m,t_{n+\frac{1}{15}}\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} U(x_m,t_n) \\ U(x_m,t_{n+\frac{1}{15}}) \end{bmatrix} \tag{17}$$

From Equation (17), the following values in Equation (18) can be obtained:

$$\left.\begin{array}{ll} Q_0(x_m,t_n)=1 & Q_1(x_m,t_n)=x_m t_n \\ Q_0(x_m,t_{n+\frac{1}{15}})=1 & Q_1(x_m,t_{n+\frac{1}{15}})=x_m t_{n+\frac{1}{15}} \end{array}\right\} \quad (18)$$

Substituting the values of Equation (18) into Equation (17), Equation (19) was generated.

$$\begin{bmatrix} 1 & x_m t_n \\ 1 & x_m t_{n+\frac{1}{15}} \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}=\begin{bmatrix} U_{m,n} \\ U_{m,n+\frac{1}{15}} \end{bmatrix} \quad (19)$$

Equation (19) was solved to obtain a_1 as:

$$a_1=\frac{15U_{m,n+\frac{1}{15}}-15U_{m,n}}{kx_m}$$

When $r=0,1,$ was substituted into Equation (14), Equation (20) was obtained

$$U(x,t)=a_0Q_0+a_1Q_1 \quad (20)$$

By substituting the values of a_1, Q_0, Q_1 in Equation (20), Equation. (21) was obtained

$$U(x,t)=a_0+15xt\frac{\left(U_{m,n+\frac{1}{15}}-U_{m,n}\right)}{kx_m} \quad (21)$$

Taken the first derivatives of Equation (21) with respect to t, Equation (22) is derived.

$$U'(x,t)=15x\left(\frac{U_{m,n+\frac{1}{15}}-U_{m,n}}{kx_m}\right) \quad (22)$$

Collocating Equation (22) at $x=x_m$ yielded

$$U'(x,t)=15\left(\frac{U_{m,n+\frac{1}{15}}-U_{m,n}}{k}\right) \quad (23)$$

But from Equation (1) it is clear that Equation (22) is equal to Equation (13), which implies that

$$15\frac{\left(U_{m,n+\frac{1}{15}}-U_{m,n}\right)}{k}=\frac{225U_{m+\frac{1}{15},n}+225U_{m-\frac{1}{15},n}-450U_{m,n}}{h^2} \quad (24)$$

Manipulating mathematically Equation (24) and putting $r=\frac{k}{h^2}$, Equation (25) is obtained

$$U_{m,n+\frac{1}{15}} = (1-30r)U_{m,n} + 15r\left(U_{m+\frac{1}{15},n} + U_{m-\frac{1}{15},n}\right) \tag{25}$$

Equation (25) is a new numerical scheme for solving the heat conduction equation.

To illustrate this method, we use it to solve problems (1) and (2) respectively.

3. Stability analysis

To find the stability condition for Equation (25), let $Mh=1$, and denote the errors at the grid points in the range $-1 \le x \le 1$, at $t=0$ by $Z(mh) = Z_m^{\,0}\ (m=0,1,2,\ldots,M)$.

But according to Ibiejugba et al. (1992), since $Z_m^{\,n}$ satisfies the original differential equation, we got

$$Z_{m,n+\frac{1}{15}} = (1-30r)Z_{m,n} + 15r\left(Z_{m+\frac{1}{15},n} + Z_{m-\frac{1}{15},n}\right) \tag{26}$$

Let the solution of the finite difference equation which reduces to $e^{i\beta x}$ be,

$$Z_m^{\,n} = e^{\alpha n k} e^{i\beta m h} \tag{27}$$

Substituting Equation (27) in Equation (26) and carrying out mathematical manipulations, Equation (28) was obtained.

$$e^{\frac{\alpha k}{15}} = 1 - 30r + 15r\left(e^{\frac{i\beta h}{15}} + e^{-\frac{i\beta h}{15}}\right) \tag{28}$$

Let $e^{\frac{\alpha k}{15}} = \xi$, then by manipulation again, Equation (29) was obtained.

$$\xi = 1 - 60r\sin^2\frac{\beta h}{30} \tag{29}$$

Equation (29) is called the amplification error of the equation.

Thus, for stability, the work of Saumaya et al. (2012) is followed. In which they suggested that the $|\xi| \le 1$, hence we have $-1 \le 1 - 60r\sin^2\frac{\beta h}{30} \le 1$, and by manipulation, the below equation is obtained.

$r\sin^2\frac{\beta h}{30} \le \frac{1}{30}$, and $r \le \frac{1}{30}$, since $\sin^2\frac{\beta h}{30} \le 1.$

And hence, since $r \le \frac{1}{30}$ the equation is conditionally stable.

4. Error analysis

To analyze the errors involved, the work of Yildiz (2001) and Zheyin (2012) and Equation (25) were considered.

$$U_{m,n+\frac{1}{15}} - (1-30r)U_{m,n} - 15r\left(U_{m+\frac{1}{15},n} + U_{m-\frac{1}{15},n}\right) = 0 \tag{30}$$

From Equation (30), Taylor's expansion of $U_{m,n+\frac{1}{15}}$, $U_{m+\frac{1}{15},n}$, $U_{m-\frac{1}{15},n}$ was found and by substitution back into equation and manipulating it mathematically the Equation (31) was obtained.

$$U_{m,n+\frac{1}{15}} - (1-30r)U_{m,n} - 15r\left(U_{m+\frac{1}{15},n} + U_{m-\frac{1}{15},n}\right) = \frac{k^2}{450}\left(\frac{\partial^2 U}{\partial t^2} - \frac{1}{90r}\frac{\partial^4 U}{\partial t^4}\right)_{m,n} + \ldots \tag{31}$$

Also, the difference Equation (25) can be manipulated mathematically to obtained,

$$\overline{U}_{m,n+\frac{1}{15}} - (1-30r)\overline{U}_{m,n} - 15r\left(\overline{U}_{m+\frac{1}{15},n} + \overline{U}_{m-\frac{1}{15},n}\right) = 0 \tag{32}$$

Subtracting Equation (32) from Equation (31) gave the error equation (Equation (33)).

$$Z_{m,n+\frac{1}{15}} - (1-30r)Z_{m,n} - 11r\left(Z_{m+\frac{1}{15},n} + Z_{m-\frac{1}{15},n}\right) = \frac{k^2}{450}\left(\frac{\partial^2 U}{\partial t^2} - \frac{1}{90r}\frac{\partial^4 U}{\partial t^4}\right)_{m,n} + \ldots \quad (33)$$

The quantity $\frac{k^2}{450}\left(\frac{\partial^2 U}{\partial t^2} - \frac{1}{90r}\frac{\partial^4 U}{\partial t^4}\right)_{m,n} + \ldots$ in Equation (33) is called the local truncation error

of the difference formula $\overline{U}_{m,n+\frac{1}{15}} = (1-30r)\overline{U}_{m,n} + 15r\left(\overline{U}_{m+\frac{1}{15},n} + \overline{U}_{m-\frac{1}{15},n}\right)$ while

$\frac{k^2}{450}\left(\frac{\partial^2 U}{\partial t^2} - \frac{1}{90r}\frac{\partial^4 U}{\partial t^4}\right)_{m,n}$ is the principal part. The method is of order $k^2 + kh^2$.

The intentions of this method were (1) to avoid the cost of solving a large system of coupled ODEs often arising from the reduction methods, and (2) to eliminate the usual draw-back of stiffness arising in the conventional reduction method by semi-discretization.

5. Results

5.1 Specific problems

Example 1: Use the scheme to approximate the solution to the heat equation.

$$\frac{\partial U}{\partial t} - \frac{\partial^2 U}{\partial x^2} = 0, \quad 0 < x < 1 \quad 0 < t$$

$$U(0,t) = U(1,t) = 0, \ t > 0$$

$$U(x,0) = \sin \pi x, \quad 0 \le x \le 1$$

Table 1: Calculated temperatures using Equation (25) on example 1

x	**Computed solution** $U(x,t)$	**Exact solution** $U(x,t)$	**Schmidt Method** $U(x,t)$	**Errors**	
				New Method	**Schmidt Method**
0	0	0	0	0	0
0.1	0.308008706	0.308002141	0.307963277	6.6 X E-6	2.1 X E-4
0.2	0.585867367	0.585854886	0.58577788	1.2 X E-5	4.0 X E-4
0.3	0.806377253	0.806360073	0.806254085	1.7 X E-5	5.6 X E-4
0.4	0.947953314	0.947932118	0.947808521	2.0 X E-5	6.6 X E-4
0.5	0.996737101	0.996715865	0.996584857	2.1 X E-5	1.2 X E -4
0.6	0.947953314	0.947932118	0.947808521	2.0 X E-5	6.6 X E-4
0.7	0.806377253	0.806360073	0.806254085	1.7 X E-5	5.6 X E-4
0.8	0.585867367	0.585854886	0.58577788	1.2 X E-5	4.0 X E-4
0.9	0.398221058	0.308002141	0.307963277	6.6 X E-6	2.1 X E-4
1.0	0	0	0	0	0

Example 2: Use the scheme to approximate the solution to the heat equation

$$\frac{\partial U}{\partial t} - \frac{\partial^2 U}{\partial x^2} = 0 \qquad 0 < t$$

$$U(-1,t) = U(1,t) = 0, \qquad t > 0$$

$$U(x,0) = \cos\left(\frac{\pi x}{2}\right), \qquad -1 \le x \le 1, t = 0$$

Table 2: Calculated temperatures using Equation (25) on example 2

x	**Exact Solution** $U(x,t)$	**Computed Solution** $U(x,t)$	**Schmidt method**	**Errors**	
				New Method	**Schmidt Method**
-1.0	0	0	0	0	0
-0.75	0.380721639	0.380741429	0.380659316	1.9 X E-5	4.2 X E-4
-0.50	0.703481860	0.703518427	0.703366704	3.7 X E-5	7.9 X E -4
-0.25	0.919143346	0.919191122	0.918992885	4.8 X E-5	1.0 X E-3
0	0.994873588	0.994925302	0.995899602	5.2 X E-5	2.3 X E- 3
0.25	0.919143346	0.911191122	0.918992885	4.8 X E-5	1.0 X E-3
0.50	0.703481860	0.703518427	0.703366704	3.7 X E-5	7.9 X E -4
0.75	0.380721639	0.380741429	0.380659316	1.9 X E-5	4.2 X E-4
1.00	0	0	0	0	0

6. Discussion

Results obtained from the action of Equation (23) on examples 1 and 2 have shown that the new off-grid method is more accurate than the known explicit Schmidt method as can be seen in Tables 1 and 2. From the error analysis, one could see that the error in the new scheme is minimal, and in terms of stability the new scheme with $r \leq \frac{1}{30}$ is more stable than the Schmidt explicit scheme which is verifiable from the results obtained from the stability analysis. Since its value of r is less than that of the known Schmidt. The numerical results have also confirmed the efficacy and validity of the new off-grid method in solving conduction equation.

7. Conclusions

A continuous interpolant is proposed for solving parabolic partial differential equation in one space variable without discretization. To check the numerical method, it is applied to solve two different test problems with known exact solutions. The results obtained are compared with the results from Schmidt method, and with the exact solutions obtained from analytic method. The numerical results confirmed the efficacy and the validity of the new numerical scheme and suggested that it is an interesting and viable numerical method which does not involve the reduction of PDE to a system of ODEs.

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