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Probabilistic Design of a Continuous One Way Reinforced Concrete Slab in Flexure

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Abstract
In this study, the probabilistic design of a continuous one way reinforced concrete slab under uncertain gravity loading is carried out in accordance with the BS 8110 design rules. The design criterion considered was flexure. The design points of the failure function corresponding to the limit state in flexure and the corresponding reliability indices were obtained using a MATLAB program developed based on First-Order reliability algorithm. The implied safety index value of 1.849 obtained at the first interior support and middle of end span fell below the target safety index value of 3.0 recommended for a reinforced concrete slab in flexure, showing the weakness of the ultimate limit state design at the first interior support and middle of end span. It was also found that the implied safety index value of 3.196 obtained at interior support and middle of interior span satisfied the target safety index of 3.0, showing that the area of tension reinforcement (377 mm^2) provided at the two locations was adequate to resist the slab load in flexure. The results of the probabilistic design carried out at predefined safety indices of 3.0, 4.0, 5.0 and 6.0 also showed that the area of tension reinforcement increased as the target safety index increased. It was also found that the areas of tension reinforcement obtained for a target safety index of 3.0 were more economical compared to the areas of tension reinforcement obtained for target safety indices of 4.0, 5.0 and 6.0 respectively for the same loading and structural geometry.

Keywords: *Probabilistic design, Singly reinforced concrete slab, Uncertain gravity loading, Design points, Failure function, Deterministic design, First Order reliability estimate*

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1. Introduction

According to Mosley and Bungey (1990), a structure must be able to serve its intended purposes and must also be capable of resisting all loads imposed on it without failure. The achievement of absolute safety of civil engineering structures still remains a goal that cannot be attained in a deterministic framework design parameters are random and stochastic in behaviour (Melchers, 1999; Ranganathan, 1999; Abubakar, et al., 2014). According to Afolayan (2002), structural problems that occur in real world are not deterministic in nature. Economical design of structures may not be possible using judgmental safety factors as the actual loads that act on a structure are not known (El-Reedy, 2013; Abubakar, 2006; Benu and Sule, 2012). The failure and eventual collapse of structures in Nigeria usually result from underestimation or negligence of uncertainties in structural engineering design models. This has destroyed lives and has caused damage to millions of properties in Nigeria in recent times worth millions of naira (Chendo and Obi, 2015). The performance of structures below expectation may be attributed to uncertainties in the structural loads' estimates, inadequate estimates of strengths of materials, variability in exposure condition, poor structural design models, poor workmanship, poor supervision of construction works, etc (John and Raji, 2015). Probabilistic design plays an important role in the safety analysis of structures as it provides a balance between safety, serviceability, and economy (Olawale et al., 2020). Abubakar and Ma'arufa (2014) carried out a reliability-based design of a two-way reinforced concrete slab to Eurocode 2 using a FORTRAN code based on First-Order reliability approach. They found that the reliability-based design approach yielded area of steel and final depth of slab which met with the target safety index value of 3.0 for flexural members. John and Raji (2015) also carried out a reliability-based design of a two-way slab designed based on BS 8110 (1997), part 1 using First Order Reliability Method coded in

FORTRAN language. They found that both the deterministic and reliability-based design methods yielded acceptable safety indices of 3.20 and 1.48 for flexure and serviceability criteria respectively. They also found that reliability-based design was about 10% economical compared to deterministic design which gave an unacceptable safety index value at short span showing that the ultimate limit state design should be reviewed at short span of a continuous reinforced concrete two-way slab. Uncertainties associated with the design parameters are accounted for using probability and statistics (Abejide, 2014; Abubakar and Edache, 2007).

In this paper, a probabilistic design of a continuous one way reinforced concrete slab designed in accordance with the BS 8110 design rules is carried out based on first-order reliability method. The failure criterion considered in the development of failure function is flexure. The design points corresponding to the failure function developed were obtained using a first order reliability algorithm coded in a MATLAB environment.

2. Development of failure function in flexure

The failure function was developed in accordance with the design provisions of BS 8110: Part 1-3, 1997, for design of concrete structures. A simply supported reinforced concrete slab under a uniform gravity loading and the assumed stress block considered in the reliability study is shown in (Fig. 1).

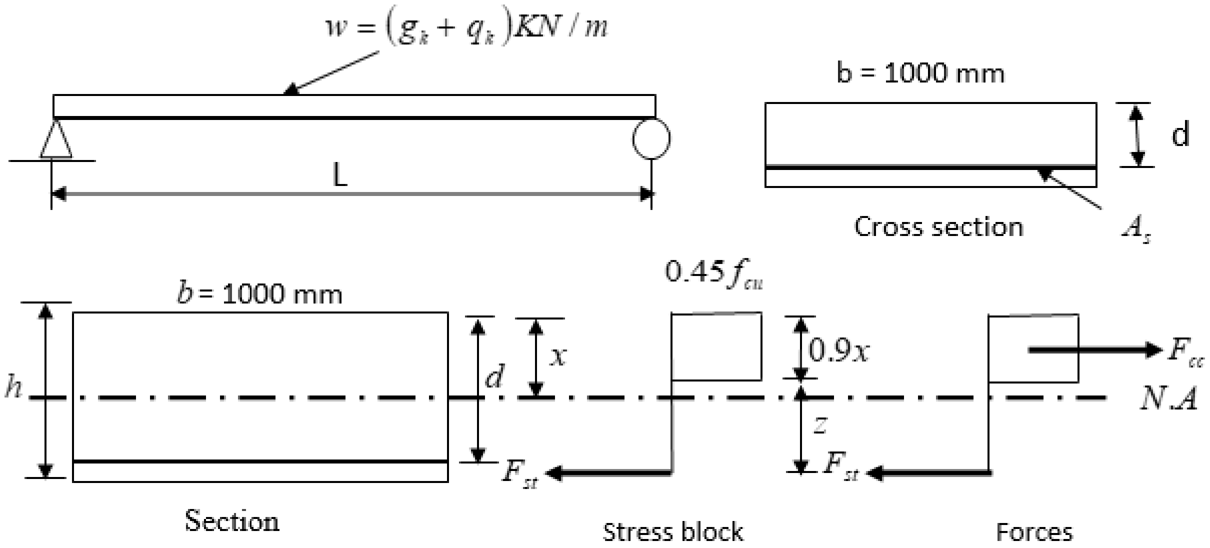

Fig. 1: A continuous one way reinforced concrete slab under uniform loading and assumed stress block

where A_s = area of tension bar, b = breadth of slab section = 1000 mm, d = effective depth of tension bar, x = depth of neutral axis, F_{cc} = compressive force of concrete, F_{st} = tensile force of steel bar

From Fig. 1,

$$F_{cc} = F_{St} \tag{1}$$

The compressive force in concrete is given by:

$$F_{cc} = 0.45 f_{cu} b * s \tag{2}$$

where s = depth of stress block

The tensile force in steel is given by:

$$F_{St} = 0.87 f_y A_s \tag{3}$$

where f_y = characteristic strength of reinforcing bar, A_s = area of tension reinforcement

Applying Equation (1),

$$0.45 f_{cu} b * s = 0.87 f_y A_s \tag{4}$$

$$s = 0.9x \tag{5}$$

Applying Equation (4),

$$s = \frac{0.87 f_y A_s}{0.45 f_{cu} b} \tag{6}$$

The moment of resistance of the section about the centroid of the compressive force is given by:

$$M_{ur} = F_{st} * z \tag{7}$$

Where z = lever arm distance representing the distance between the centroids of the compressive and tensile force respectively

The lever arm distance z is given by:

$$z = d - s/2 \tag{8}$$

Applying Equations (3), (6) and (8) changes Equation (7) to:

$$M_{ur} = 0.87 f_y A_s \left(d - \frac{0.87 f_y A_s}{0.9 f_{cu} b} \right) \quad (9)$$

Equation (9) represents the ultimate moment of resistance of a singly reinforced concrete beam in flexure.

2.1 Flexure criterion

The failure function in flexure is given by:

$$g(x) = M_{ur} - M_{app} \quad (10)$$

where M_{ult} = ultimate moment of resistance of the beam, M_{app} = gravity load induced moment.

The maximum bending moment due to applied load is given by:

$$M_{app} = \alpha_m F L \quad (11)$$

where α_m = moment coefficient obtained from BS 8110, F = uniformly distributed gravity load on beam at ultimate limit state, L = beam span.

The uniformly distributed gravity load on the beam at ultimate limit state is given by:

$$F = (1.4 g_k + 1.6 q_k) * sp \quad (12)$$

where sp = spacing of beams, g_k = dead load on the beam, q_k = imposed load on the beam

Equation (12) can be written in terms of load ratio and beam spacing as:

$$F = q_k (1.4\alpha + 1.6) * sp \quad (13)$$

where α = load ratio

The load ratio α is given by:

$$\alpha = g_k / q_k \quad (14)$$

Applying Equation (13), Equation (11) becomes:

$$M_{app} = \alpha_m q_k (1.4\alpha + 1.6) * sp * L \quad (15)$$

Applying Equation (9) and Equation (15), Equation (10) becomes:

$$g(x) = 0.87 f_y A_s \left(d - \frac{0.87 f_y A_s}{0.9 f_{cu} b} \right) - \alpha_m q_k (1.4\alpha + 1.6) * sp * L \quad (16)$$

For $sp = L$, Equation (16) becomes:

$$g(x) = 0.87 f_y A_s \left(d - \frac{0.87 f_y A_s}{0.9 f_{cu} b} \right) - \alpha_m q_k (1.4\alpha + 1.6) * L^2 \quad (17)$$

Equation (17) is the limit state function for a continuous one way reinforced concrete slab in flexure. The limiting value of lever arm distance z is given by:

$$z = d\left(0.5 + \sqrt{0.25 - 1.111k}\right) \le 0.95d \quad (18)$$

3. Materials and methods

3.1 First order reliability method

According to First Order Reliability Method, the vector

$$x = (x_1, x_2, \ldots, x_n)^T \quad (19)$$

is the vector of uncertain variables whose joint probability function is given by:

$$F_X(X) = P\left(\bigcap_{i=1}^{n} \{X_i \le x_i\} \right) \quad (20)$$

$F_X(X)$ is at least continuous and locally differentiable with respect to the uncertain parameters. This implies that the probability densities of $F_X(X)$ exist. The failure function, $g(x)$ of a structure at a limit state is usually a function of uncertain basic variables which are random and stochastic in character. Mathematically, $g(x)$ is defined such that $g(x) > 0$ corresponds to the safe state of the structure or structural element, $g(x) < 0$ corresponds to the failure state of the structure and $g(x) = 0$ is limiting state surface. This line is the demarcation between the safe and unsafe state of the structure. The First Order Reliability estimate approximates the probability of failure of structure as a monotone function of reliability index as:

$$p_f = P[g(x) \le 0] = \phi(-\beta) \quad (21)$$

where β = geometric reliability index and it represents the minimum distance between the origin and the failure surface, $\phi(.)$ = standard normal integral. The geometric index β is defined as:

$$\beta = \min \{\|x\|\} \; for\, (x : g(x) < 0) \quad (22)$$

The values of the basic variables that minimize the distance between the origin and the failure surface subject to $g(x) = 0$ are obtained by optimization.

The design is considered satisfactory if:

$$\beta_{Implied} \approx \beta_{Target} \quad (23)$$

where $\beta_{Implied}$ = implied safety index obtained from reliability analysis using a MATLAB code and β_{Target} = target safety index (JCSS, 2006).

The probabilistic models of the uncertain variables and their corresponding probability distributions are presented in Table 1.

Table 1: Probabilistic models of the basic variables

Variables	Expected value (X)	Standard Deviation (X)	Coefficient of Variation (X)	Type of Probability Distribution
q_k	3.0 KN/m^2	0.9 KN/m^2	0.30	Gumbel
d	209 mm	10.45 mm	0.05	Normal
L	8000 mm	400 mm	0.05	Lognormal
fy	460 N/mm^2	69 N/mm^2	0.15	Lognormal
fcu	30 N/mm^2	4.5 N/mm^2	0.15	Lognormal
α	2.4	-	-	Fixed value
b	1000 mm	-	-	Fixed value

Source: (Ranganathan, 1999; Melchers, 1999; Nader, 2017; Goutham and Manjunath, 2016; Abejide, 2014; Ravindra and Manjunath, 2010)

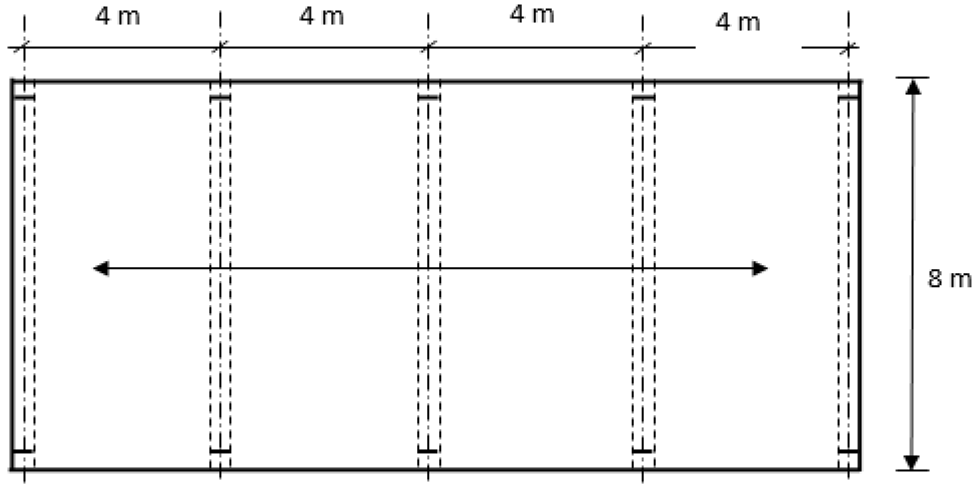

Fig. 1: A continuous one way reinforced concrete slab

3.2 Reliability analysis

The deterministic design of a four span continuous one way reinforced concrete slab was carried out according to BS 8110 design rules. The results obtained based on BS 8110 design codes were compared with the results obtained from probabilistic design for a target safety index of 3.0. The dead load due to floor finishes and ceiling is 1.5KN/m^2 and the centre to centre spacing of the beams is 4m. The imposed floor load is 3.0 KN/m^2.

4. Results

The results of the probabilistic design of a four span continuous one way reinforced concrete solid slab carried out based on BS 8110 and those obtained from probabilistic design are presented in tables 1. The results of probabilistic design carried out based on First-Order Reliability method for target safety indices of 3.0, 4.0, 5.0 and 6.0, for first interior support, middle of end span, interior support and middle of interior span are presented in tables 2 and 3 respectively.

Table 1: Comparison of results of deterministic and probabilistic design

Design Location	Deterministic Design based on BS 8110	Implied Safety Index	Target Safety Index = 3.000	
			Probabilistic Design	Implied Safety Index
First Interior Support	As = 377 mm^2 d = 209 mm	$\beta = 1.845$ (Not satisfactory)	As = 500 mm^2 d = 203 mm	$\beta = 2.993$
Interior Support	As = 377 mm^2 d = 209 mm	$\beta = 3.192$ (Satisfactory)	As = 377 mm^2 d = 203 mm	$\beta = 3.196$
Middle of Interior Span	As = 377 mm^2 d = 209 mm	$\beta = 3.192$ (Satisfactory)	As = 377 mm^2 d = 203 mm	$\beta = 3.196$
Middle of End Span	As = 377 mm^2 d = 209 mm	$\beta = 1.845$ (Not satisfactory)	As = 500 mm^2 d = 203 mm	$\beta = 2.993$

Table 2: Results of probabilistic design for different target safety indices for first interior support and middle of end span

Target Safety Index = 3.00	Target Safety Index = 4.00	Target Safety Index = 5.00	Target Safety Index = 6.00
As = 500 mm^2 d = 203 mm $\beta_{Implied} = 2.993$	As = 688 mm^2 d = 203 mm $\beta_{Implied} = 3.999$	As = 1085 mm^2 d = 204.3 mm $\beta_{Implied} = 4.993$	As = 2650 mm^2 d = 207 mm $\beta_{Implied} = 5.995$

Table 3: Results of probabilistic design for different target safety indices for interior support and middle of interior span

Target Safety Index = 3.00	Target Safety Index = 4.00	Target Safety Index = 5.00	Target Safety Index = 6.00
As = 377 mm^2 d = 203 mm $\beta_{Implied} = 3.196$	As = 490 mm^2 d = 203 mm $\beta_{Implied} = 3.997$	As = 770 mm^2 d = 204.3 mm $\beta_{Implied} = 4.985$	As = 1900 mm^2 d = 207 mm $\beta_{Implied} = 5.999$

5. Discussion of results

Based on the results obtained from Tables 1, 2 and 3, it can be observed that:

i. The implied safety index value of 1.849 obtained at first interior support and middle of end span falls below the target safety index value of 3.0 as recommended by JCSS (2006), showing that the deterministic design is not satisfactory and needs to be reviewed to achieve a better performance.

ii. The implied safety index value of 3.196 obtained at interior support and middle of interior span satisfies the target safety index value of 3.0 (JCSS, 2006) showing that the deterministic design is satisfactory.

iii. The area of tension reinforcement increases with increase in value of target safety index. This is because an increase in area of tension reinforcement increases the moment capacity of the beam to resist more bending moment. This implies that reliability index of the beam increases with increase in moment capacity.

iv. The effective depth of reinforced concrete slab remains constant at 203mm for target safety indices of 3.0, 4.0 and 5.0 respectively, but increases to 207 mm for a target safety index of 6.0. The constant value of the effective depth at target safety indices of 3.0, 4.0 and 5.0 may be attributed to the fact that the failure surface at target safety index of 3.0 coincided with those of 4.0 and 5.0 respectively. The marginal increase in the value of effective depth may be due to a target safety index of 6.0 generating a different failure surface (Ranganathan, 1999).

v. The probabilistic design gives an effective depth value of 203 mm for a target safety index of 3.0 (JCSS, 2006) while the effective depth value obtained based on BS 8110 is 209 mm. This yields about 2.9% savings in materials.

vi. From Tables 2 and 3, it can be seen that the areas of tension reinforcement obtained for a target safety index of 3.0 are more economical compared to the areas of tension reinforcement obtained for target safety indices of 4.0, 5.0 and 6.0 respectively. This is in agreement with Abubakar and Ma'arufa (2014), that the results obtained when a target safety index of 3.0 is considered are cheaper compared to those of the target safety index of 4.0 for the same loading and structural geometry.

6. Conclusions

The results of probabilistic design of a four span continuous reinforced concrete slab carried out based on First Order reliability method have been presented. It was found that the implied safety index value of 1.849 obtained at first interior support and middle of end span is not satisfactory and should be reviewed to enhance a better performance as it fell below the target safety index value of 3.0 recommended by JCSS (2006). It was also found that the implied safety index of 3.196 obtained at interior support and middle of interior span satisfied the target safety index value of 3.0. The area of tension steel was also found to increase as the value of the target safety index increased. It was also found that the effective depth of reinforced concrete slab remained the same

(203mm) for target safety indices of 3.0, 4.0 and 5.0 respectively, but increased (207 mm) for a target safety index of 6.0. The probabilistic design gave an effective depth value of 203 mm for a target safety index of 3.0 while the effective depth value obtained based on BS 8110 was 209 mm. This yielded about 2.9% savings in construction materials. It was also found that the areas of tension reinforcement obtained for a target safety index of 3.0 are more economical compared to the areas of tension reinforcement obtained for target safety indices of 4.0, 5.0 and 6.0 respectively for the same loading and structural geometry.

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