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Modelling and Evaluating the Reliability of Programmable Logic Controller Using the Wiener Process

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Abstract
Programmable Logic Controllers (PLCs) are used to make logic-based decisions in automated industrial system, and they are able to operate on a wide range of industrial equipment. Despite their robustness, "design for longevity" idea, and tolerance to adverse operational issues, which include unclean air, humidity, vibration, electrical noise, and so on, PLC-based control systems, however, can still fail, resulting in significant downtime. In this paper, the Wiener process, which has been used in numerous reliability analyses and evaluations of systems, has been implemented for the reliability evaluation and management of a PLC system. Results from the evaluation show that the Wiener process and the Copula function were effective in the reliability analysis of the PLC and were able to effectively describe the randomness and coupling correlation of the PLC degradation process. The value of Pearson increased from approximately 0.39 to 1 and then dropped to -0.76, while the scatter plot for the Weiner process has a positive (+ve) correlation. The results from the study show a relationship between the variables, and was achieved through their implementation in a Matlab simulation platform that resulted in concordance between the variables when the copula function was used.
Keywords: *Programmable Logic Controllers, Reliability analysis and evaluation, Clustering, Copula function, Wiener Process, Concordance*

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1. Introduction

With the increased involvement of product end users in the design, development and lifecycle management of manufactured products, early product reliability evaluation has come to receive a lot of attention from reliability engineering scholars. Due to the rapid advancement of science and technology and the complexity in product designs and their architecture, product reliability evaluation and management has become increasingly difficult to achieve with the many diversities in the failure modes of most of the products and the small amount of failure data available for life-long products evaluation.

In the last ten years, performance degradation-based reliability technology has grown in popularity, it has widely been used in reliability evaluation and has become an important research direction in reliability engineering field (Pan et al., 2020). Reliability is the probability that an item can perform its intended function for a specified interval under stated conditions following prescribed procedures. Due to the increasing reliability of systems or goods in applications, failure data are harder to gather in laboratory and in the fields.

As a result, system reliability assessments and evaluations in recent time depend now more on, the degrading performances data. Also, a variety of sensors are now been used to practically collect information about how a system(s) or items are degrading. Therefore, it has become common practice to study the deteriorating systems using data-driven degradation models. The general route models and the stochastic process models, such the Wiener process and Gamma process, are now frequently been utilized among these degradation models (Gao et al., 2018).

Kim & Kolarik (1992) proposed the concept of performance reliability to address the problem of predicting product reliability in real time, while Wang et al., (2020) developed a new model to extend the Wiener degradation process's inference capabilities with random effects, this model which have the advantage of been used in analyzing degradation data when the sample sizes are small is used in taking measurements where errors are taking into account. Pan et al., (2018) proposed a reliability estimation approach that is based on the expectation maximization algorithm and Wiener processes, they used the expectation-maximization algorithm to estimate the model parameters effectively.

The majority of the preceding literature focuses solely on the reliability analysis of single performance parameters. However, many of the modern complex products, frequently have performance degradation that requires comprehensive reliability consideration and evaluations. Many studies have also been conducted in order to solve the problem of multi-performance degradation. Among them include, Cheng et al., (2019) who proposed a new method for addressing the problem of degradation modeling and reliability prediction of machinery with multiple degradation characteristics by combining the double-Wiener process model with the Monte Carlo algorithm.

Bansal & Cheung, (2017), proposed a new stochastic simulation-based approach for evaluating multiple failure probability curves in a reliability problem with multiple performance functions. Zhang et al., (2019) developed a degradation-based continuous multistate reliability model based on the state function definition, which provides a new way to analyze reliability in the case of multiple degradation processes. In order to accurately analyze the reliability of a system, a multi-state reliability analysis method based on performance degradation and the universal generating function was proposed by Linjie et al., (2017). Warm standby is another energy-saving redundancy method that uses less energy than traditional hot standby.

To improve the reliability of a system, it can be naturally integrated with an energy storage technique. However, the integration of both techniques and the benefits it provides to the reliability of a system have not been fully reported in literature. Although attempt have been made by Heping et al., (2021) to address this limitation, by developing a reliability model for demand-based warm standby systems with capacity storage. Where the chronological characteristics of the warm standby components were explicitly explored before and after their activation in the model. PLCs have long been employed in automated industrial operations to make logic-based decisions, and they are capable of operating on a wide range of industrial equipment as well as full automated systems. PLC-based control systems can nevertheless fail, by resulting in a considerable downtime, despite their toughness, design for longevity concept, and resistance to their adverse surroundings which include, dirty air, humidity, vibration, electrical noise etc. which are prevalent in factories and industries.

With these characteristics in mind, as well as the PLC's key role and use in industries. The PLC's reliability level must be maintained at a high level on a regular basis because a single failure could have catastrophic repercussions with large costs and serious safety risks. In this paper, a wiener process have been proposed and deployed for reliability analysis and evaluation of the PLC system of an industrial machine (automatic sieving machine). The wiener process which allows for a robust and efficient evaluation of the system, also addresses the deterioration of the systems using data-driven approach, by generating random variables and data that is used by the copula function model of the process for the evaluation of the reliability of the PLC system.

2. Wiener process

The Wiener process which has been used in several reliability analysis and evaluation of systems was applied for the reliability evaluation of the PLC system of an industrial machine. The wiener process which allows for a robust and efficient evaluation of the system, was used to address the deterioration of the PLC, using clustering and data-driven approach, by generating random variables and data which was used by the copula function model of the process to evaluate the reliability of the PLC system using Matlab codes. The implementation of the codes was achieved by using 1000 data samples, for reproducibility, the random number generator was configured to the default wiener process. Random numbers were generated, as well as gamma-inverse, beta-inverse, and t-inverse distribution functions for the variables. The mean and standard deviation of the distribution of the variables were also obtained.

The Wiener process is among the most widely used stochastic processes for degradation models and has received a great deal of attention, with research focusing on topics such as confidence intervals (CIs), parameter estimation and reliability modelling. The Wiener process can provide a more accurate description of the system's dynamic characteristics. First, we must determine the connection between the acceleration variable and the degradation model in order to analyze accelerated degradation data. In general, there are two types of relationships.

The drift parameter is affected by the acceleration variable, whereas the diffusion parameter is constant. The other connection is that the drift and diffusion parameters are all affected by the acceleration variable. The degradation mechanism of products from the same batch should be the same. It is assumed that the degradation data can be described using the Wiener process. Let $X(t)$ be a standard Wiener process, which has the following basic properties.

(i) $X(0) = 0$ and $X(t)$ is continuous when $t = 0$.

(ii) $X(t) \mid t \geq 0$ has a stable independent increment,

(iii) $X(t_2) - X(t_1)$ and $X(t_4) - X(t_3)$ are independent of each other in any two disjoint time intervals $[t_1, t_2]$ and $[t_3, t_4]$.

For each increment of the time interval, $\Delta X = X(t + \Delta t) - X(t)$, obeys the $N(\mu\Delta t, \sigma^2\Delta t^2)$ normal distribution. These properties, can be used in the reliability modelling of the PLC system with single and multiple performance parameter(s).

2.1 Reliability modelling of single performance parameters for the PLC system

Let's assume there are n performance parameters for the PLC. As the PLC's usage time increases, its performance degrades. At time, t, the degradation amount $X_i(t)$ of the ith performance parameter of the PLC is a random variable subject to a certain distribution, such that the process $\{X_i(t), t \geqslant 0\}$ is random. It is assumed that the performance degradation amounts $X_i(t)$ of the PLC can be represented using the Wiener process.

$$X_i(t) = \delta_i t + \beta_i X_{0i}(t) \quad (1)$$

where the parameter δ_i is the drift parameter in the ith performance parameter degradation, β_i is the diffusion parameter in the ith performance parameter degradation, $X_{0i}(t)$ is the standard Wiener process of the ith performance parameter, and $E[X_i(t)] - \delta_i t, D[X_i(t)] = \beta_i^2 t$.

If we assume that the PLC's performance degradation $X_i(t)$ is subjected to some form of univariate or multiple Wiener process $\{X_i(t), t \geqslant 0\}$, and the failure threshold is $h_i(h_i > 0)$, according to the definition of degradation failure, the PLC lifetime T is the time corresponding to the degradation of the i performance parameter of the n degradation $X_i(t)$ $(i = 1, 2, \dots, n)$ first reaching the failure threshold h_i.

$$T_i = inf\{t | X_i(t) \geq h_i\} \quad (2)$$

According to Pan et al., (2020), the product's single performance parameter's reliability is expressed as follows:.

$$R_i(t) = \phi\left(\frac{h_i - \delta_i t_i}{\beta_i\sqrt{t_i}}\right) - exp\left(\frac{2\delta_i h_i}{\beta_i^2}\right)\phi\left(\frac{-\delta_i t_i - h_i}{\beta_i\sqrt{t_i}}\right) \quad (3)$$

where the parameter δ_i is the drift parameter in the ith performance parameter degradation, β_i is the diffusion parameter in the ith performance parameter degradation, h_i is the failure threshold, t_i is the time parameter in the ith performance parameter degradation.

2.2 Modelling the reliability of multiple performance parameters

Assuming that the PLC has a n performance parameters and with some correlation. At time t, therefore, the performance degradation trajectory of the system can be expressed as, $X(t) = (X_1(t), X_2(t), \dots, X_n(t))$, such that the corresponding degradation failure threshold could be expressed as, $l = (l_1, l_2, \dots, l_n)$. Similarly, if the study assume that the joint distribution function can be expressed as $V(t_1, t_2, \dots, t_n)$, therefore the product lifetime can also be expressed as $T_1, T_2, \dots, T_n$, which is in agreement with the definition of Copula function. The Copula function C, satisfies the following formula.

$$V(t_1, t_2, \dots, t_n) = C(F_1(t_1), F_2(t_2), \cdots, F_n(t_n); \theta) \quad (4)$$

where θ is the parameter of the Copula function. The reliability of the PLC can therefore be expressed using either of Equations 5-9.

$$R(t) = P(T > t) \quad (5)$$

$$R(t) = P(T_1 > t, T_2 > t, \dots, T_n > t) \quad (6)$$

$$R(t) = 1 - \sum_{i=1}^{n} P(T_i \leq t) + \sum_{1\leq i<j\leq n} P(T_i \leq t, T_j \leq t) + \cdots$$

$$+(-1)^k \times \sum_{1\le i_1<i_2<\cdots<i_k\le n} P\left(T_{i_1} \le t, T_{i_2} \le t, \dots, T_{i_k} \le t\right) + \cdots + (-1)^n P\left(T_{i_1} \le t, T_{i_2} \le t, \dots, T_{i_k} \le t\right) \quad (7)$$

$$R(t) = 1 - \sum_{i=1}^{n} F_i(t) + (-1)^k \times \sum_{1\le i_1<i_2<\cdots<i_k\le n} C\left(F_{i_1}(t), F_{i_2}(t), \cdots, F_{i_k}(t); \theta\right) \quad (8)$$

$$R(t) = 1 - \sum_{i=1}^{n} F_i(t) + (-1)^k \times \sum_{1\le i_1<i_2<\cdots<i_k\le n} C^n\left(F_{i_1}(t), F_{i_2}(t), \cdots, F_{i_k}(t), 1,1,\dots,1; \theta\right) \quad (9)$$

where $C^n(\cdot)$ represents an n dimensional Copula function C and $2 \leqslant k \leqslant n$

2.3 Copula function

According to Nelsen, (2006), the Sklar's theorem states that a joint distribution can be decomposed into k edge distributions and a Copula function which describes the correlation between the different variables. Let V be an n dimensional distribution function, if its edge distribution is given as $F_1(x_1), F_2(x_2), \cdots, F_n(x_n)$. Therefore, to achieve the n dimensional Copula function C, the following expression must be satisfied.

$$V(x_1, x_2, \dots, x_3) = C(F_1(x_1), F_2(x_2), \cdots, F_n(x_n)) \quad (10)$$

The Copula function described here can be used to determine the joint distribution function for each edge distribution function, as long as the proper Copula function C is established. There are different families of the Copula function, and they have been discussed extensively by Nelsen, (2006) and Joe, (1997). Some of the family members include, the Elliptical family which includes both the Gaussian and the *t* Copula model. The Archimedean family also includes the Clayton's, Frank's and Gumbel's Copula models. The Archimedean family for the the Copula functions, is more useful in empirical modeling and this is due to its ease of derivation as discussed in DiLascio & Giannerini, (2012). In this paper however, the focus will be on the Frank's copula model, and their derivative.

$$C(u_1, \dots, u_K) = \frac{1}{\theta_2} \ln\left\{1 + \frac{\prod_{k=1}^{K}\left(e^{-\theta_2 u_k} - 1\right)}{\left(e^{-\theta_2} - 1\right)^{K-1}}\right\}, \quad (11)$$

where $\theta_2 \in (0,\infty)$ and $K \geq 3$; independence is attained as θ_2 reaches zero;

Clayton's copula

$$C(u_1, \dots, u_k, \dots u_K) = \left[\sum_{k=1}^{K} u_k^{-\theta_2} - K + 1\right]^{-\frac{1}{\theta_2}}, \quad (12)$$

where the parameter θ_2 is restricted on the region $(0, \infty)$ and as it approaches zero the marginal distributions become independent.

Gaussian copula

$$C(u_1, \dots, u_K) = \Phi_K[\Phi^{-1}(u_1), \dots, \Phi^{-k}(u_k), \dots, \Phi^{-K}(u_K); \theta_2], \quad (13)$$

where Φ is the cumulative distribution function of the standard univariate normal distribution and Φ_K is the standard K–variate normal distribution with correlation parameter θ_2 restricted to the interval $(-1, 1)$;

The normal or Gaussian copula is symmetric and adaptable, allowing for equal amounts of positive and negative dependence. Unlike Clayton's and Gumbel's copula models, Frank's copula is (permutation) symmetric, its tails are lighter than those of a Gaussian copula, and it only allows negative dependence for bivariate joint distributions. Clayton's copula, on the other hand, is asymmetric, with strong left tail dependence and relatively weak right tail dependence, and it is incapable of accounting for negative dependence. The computation of Frank's and Clayton's dependence parameters from association measures such as Kendall and Spearman is simple, and the relationship between them is one-to-one (Cherubini et al., 2013). Furthermore, all three copula models are comprehensive, allowing for the greatest possible range of dependence. These factors, along with the ability to describe a multivariate complex dependence using a single parameter, account for the popularity of such families in the applied literature on copulas. Generally speaking, a pair of random variables are concordant if "large" values of one tend to be associated with "large" values of the other and "small" values of one with "small" values of the other. To be more precise, let (x_i, y_i) *and* (x_j, y_j) denote two observations from a vector (x, y) of continuous random variables. We say that (x_i, y_i) *and* (x_j, y_j) are concordant if $x_i < x_j$ *and* $y_i < y_j$, or if $x_i > x_j$ *and* $y_i > y_j$. Similarly, we say that (x_i, y_i) *and* (x_j, y_j) are discordant if $x_i < x_j$ *and* $y_i > y_j$ or if $x_i > x_j$ *and* $y_i < y_j$.

Note the alternate formulation: (x_i, y_i) *and* (x_j, y_j) are concordant if $(x_i - y_i)(x_j - y_j) > 0$ and discordant if $(x_i - y_i)(x_j - y_j) < 0$ (Nelsen, 2006). The following definition of concordance applies to the sample form of the association measure known as Kendall's tau which is used to understand the strength of the relationship between two variables (Kruskall, 1958; Hollander & Wolfe, 1973; Lehmann, 1975): Let $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ denote a random sample of n observations from a vector (X, Y) of continuous random variables. There are $\binom{n}{2}$ distinct pairs (x_i, y_i) *and* (x_j, y_j) of observations in the sample, and each pair is either concordant or discordant—let *c* denote the number of concordant pairs and d the number of discordant pairs. Then Kendall's tau for the sample is defined as

$$t = \frac{c-d}{c+d} = {(c-d)}\Big/{\binom{n}{2}} \tag{14}$$

Alternatively, t is the probability of concordance less the chance of discordance for the randomly selected pairs of observations (x_i, y_i) *and* (x_j, y_j) Similar definitions apply to the population variant of Kendall's tau for a vector *(X,Y)* of continuous random variables with joint distribution function H. Let (X_i, Y_i) *and* (X_j, Y_j) be independent and identically distributed random vectors with joint distribution functions H. The chance of concordance minus the likelihood of discordance is then used to define the population form of Kendall's tau:

$$\tau = \tau_{XY} = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0] \tag{15}$$

The population version of the measure of association known as Spearman's rho is founded on concordance and discordance, just like Kendall's tau. To get this measure's population-based variant, (Kruskall, 1958:Lehmann, 1966), let $(X_1, Y_1), (X_2, Y_2)$ *and* (X_3, Y_3) be three independent random vectors with a common joint distribution function H (whose margins are again F and G) and copula C. The population version $\rho_{X,Y}$, of Spearman's rho is defined to be proportional to the probability of concordance minus the probability of discordance for the two vectors (X_1, Y_1) *and* (X_2, Y_3)—i.e., a pair of vectors with the same margins, but one vector has distribution function H, while the components of the other are independent:

$$\rho_{X,Y} = 3(P[(X_1 - X_2)(Y_1 - Y_3) > 0] - P[(X_1 - X_2)(Y_1 - Y_3) < 0]) \tag{16}$$

(the pair (X_3, Y_2) could be used equally as well). Note that while the joint distribution function of (X_1, Y_1) is $H(x, y)$, the joint distribution function of (X_2, Y_3) is $F(x)G(y)$ (because X_2 and $Y_3$are independent).

2.4 Parameter estimation with the model

Assuming that the total number of products is M, then the product performance data is measured in K times. Each performance test of the same batch of products has the same time, and the first test is set to the initial time $t_0$with the subsequent test time being t_k, $k = 1, 2, \ldots, K$. The degradation of the i performance parameter of the m product at initial time t_0 is X_m, i(t_0) = 0. X_m, $i(t_k)$ represents the degradation of the performance of the i performance parameter of the m product at time t_k. Then the degradation of the i performance parameter of the m product in the time interval $[t_k-1, t_k]$ is shown as follows.

$$\Delta X_{m,i}(t_k) = X_{m,i}(t_k) - X_{m,i}(t_{k-1}) \tag{17}$$

Due to the nature of the Wiener process, the following formula is obtained.

$$\Delta X_{m,i}(t_k) \propto N(\mu_i \Delta t_k, \sigma_i^2 \ \Delta t^2) \tag{18}$$

The probability density function of the incremental performance degradation can be expressed as follows.

$$f\left(\Delta X_{m,i}(t_k)\right) = \frac{1}{\sqrt{2\pi\sigma_i^2 \Delta t_k}} exp\left[\frac{\left(\Delta X_{m,i}(t_k) - \mu_i \Delta t_k\right)^2}{2\sigma_i^2 \Delta t_k}\right] \tag{19}$$

Its maximum likelihood function is as follows.

$$L(\mu_i, \sigma_i) = \prod_{m-1}^{M} \prod_{k=1}^{K} \frac{1}{\sqrt{2\pi\sigma_i^2 \Delta t_k}} exp\left[-\frac{\left(\Delta X_{m,i} - \mu_i \Delta t_k\right)^2}{2\sigma_i^2 \Delta t_k}\right] \quad (20)$$

Take the logarithm of both sides and find the partial derivatives of $\mu_i \; and \; \sigma_i$ respectively to obtain the estimated parameters of the model.

$$\hat{\mu}_i = \frac{\sum_{m=1}^{M} X_{m,i}}{\sum_{k=1}^{K} t_k} \quad (21)$$

$$\hat{\sigma}_i^2 = \frac{1}{M}\left[\sum_{m=1}^{M} \sum_{k=1}^{K} \frac{\left(\Delta X_{m,i}\right)^2}{\Delta t_k} - \frac{\left(\sum_{m=1}^{M} X_{m,i}\right)^2}{\sum_{k=1}^{K} t_k}\right] \quad (22)$$

According to the obtained parameter estimation values $\hat{\mu}_i$ and $\hat{\sigma}_i^2$, the edge distribution function $F_1(t_1), F_2(t_2), \cdots, F_n(t_n)$ is calculated as an input in the Copula function.

$$L(\theta) = \sum_{m=1}^{M} \sum_{k=1}^{K} ln \; C(F_1(t_1), F_2(t_2), \cdots, F_n(t_n); \theta) \quad (23)$$

where $C(F_1(t_1), F_2(t_2), \cdots, F_n(t_n); \theta)$ is the probability density function of the multidimensional

Copula function

The algorithm for this paper, has the following steps, such as specifying a number of random parameters, to use for the model, this will be followed by Specifying the linear correlation parameters, Rho of the variables, after that the model will be made to be reproducible by setting rng to default, after that a Gaussian Copula to generate matrix of n random dependent variables based on Rho, this is followed by transforming each column of the generated dependent variables into statistical distributions and get the respective inverse cumulative distribution functions. In this research the first variable was transformed using Gamma distribution, the second to Geta distribution and the third to a *t* distribution. This is followed by generating plots of the inverse cumulative distribution of the Weiner (random) data generated using the linear correlation parameter of the variables, this is followed by calculating the theoretical tau, followed by computing of the kendall and pearson correlation coefficients, and finally computing of the Gaussian coefficient of each copula

3. Results and discussion

A 3D plot of the variables was created by using plot 3, different comparative matrix plots were created such as; variable 1 against variable 2, variable 2 against variable 3, variable 1 against variable 3. The Unique deterioration data, the Kendall tau and Pearson correlation coefficient were computed, the Gaussian rank coefficient for each of the copula was obtained. Each variable's probability density function was generated and plotted. The plots of the results were obtained. Comparison plots for the various copulas were made for the first, second and third variable. A comparative matrix plot was created. The following results and plots show how effective these methods are in regards to reliability evaluation and analysis of a PLC system. The Mean and standard deviation of the weiner variables are shown in Table 1,

Table 1: Mean and Standard deviation of wiener variables

Variable	Mean	Standard deviation
X	1.9573	1.4385
Y	0.5040	0.2215
Z	-0.0576	1.3010

Copula function results

The concordance relationship is employed to understand the strength between a pair of random variables. From Table 2, we have 3 variables, which are labelled as X, Y and Z. So each row represents a variable. From the Unique deterioration data we can see that, it is not concordant when analyzing $X_{ij} and \; Y_{ij}$, but is concordant when analyzing $X_{jk} and \; Y_{jk}$ because $X_i > X_j \; and \; Y_i < Y_j$, while $X_j > X_k \; and \; Y_j > Y_k$. It is also concordant when analyzing $X_{ij} and \; Z_{ij}$, but not concordant with $X_{jk} and \; Z_{jk}$, because $X_i > X_j \; and \; Z_i > Z_j$, while $X_j > X_k \; and \; Z_j < Z_k$.

Table 2: Concordance relationship strength between random variables

	Unique deterioration data			Kendall tau			Pearson correlation coefficient			Gaussian rank coefficient		
	i	j	k	l	m	n	o	p	q	r	s	t
X	1.00	0.26	0.13	1.00	0.26	0.14	1.00	0.35	0.21	1.00	0.26	0.13
Y	0.26	1.00	-0.59	0.26	1.00	-0.58	0.35	1.00	-0.76	0.26	1.00	-0.59
Z	0.128	-0.59	1.00	0.14	-0.58	1.00	0.21	-0.76	1.00	0.13	-0.59	1.00

Kendall tau is not concordant when analyzing $X_{lm} and\ Y_{lm}$, but it is concordant when analyzing $X_{mn} and\ Y_{mn}$ because $X_l > X_m\ and\ Y_l < Y_m$, while $X_m > X_n\ and\ Y_m > Y_n$. It is also concordant when analyzing $X_{lm} and\ Z_{lm}$, but not concordant with $X_{mn} and\ Z_{mn}$, because $X_l > X_m\ and\ Z_l > Z_m$, while $X_m > X_n\ and\ Z_m < Z_n$.

The Pearson correlation coefficient is not concordant when analyzing $X_{op} and\ Y_{op}$, but is concordant when analyzing $X_{pq} and\ Y_{pq}$ because $X_o > X_p\ and\ Y_o < Y_p$, while $X_p > X_q\ and\ Y_p > Y_q$. It is also concordant when analyzing $X_{op} and\ Z_{op}$, but not concordant with $X_{pq} and\ Z_{pq}$, because $X_o > X_p\ and\ Z_o > Z_p$, while $X_p > X_q\ and\ Z_p < Z_q$.

The Gaussian rank coefficient, is not discordant when analyzing $X_{rs} and\ Y_{rs}$, but is concordant when analyzing $X_{st} and\ Y_{st}$ because $X_r > X_s\ and\ Y_r < Y_s$, while $X_s > X_t\ and\ Y_s > Y_t$ or we can say that $(X_r - X_s)\ (Y_r - Y_s) < 0$, while $(X_s - X_t)(Y_s - Y_t) > 0$. It is also concordant when analyzing $X_{rs} and\ Z_{rs}$, but not concordant with $X_{st} and\ Z_{st}$, because $X_r > X_s\ and\ Z_r > Z_s$, while $X_s > X_t\ and\ Z_s < Z_t$. From our observation it can be seen that Unique deterioration data values = Gaussian rank coefficient values.

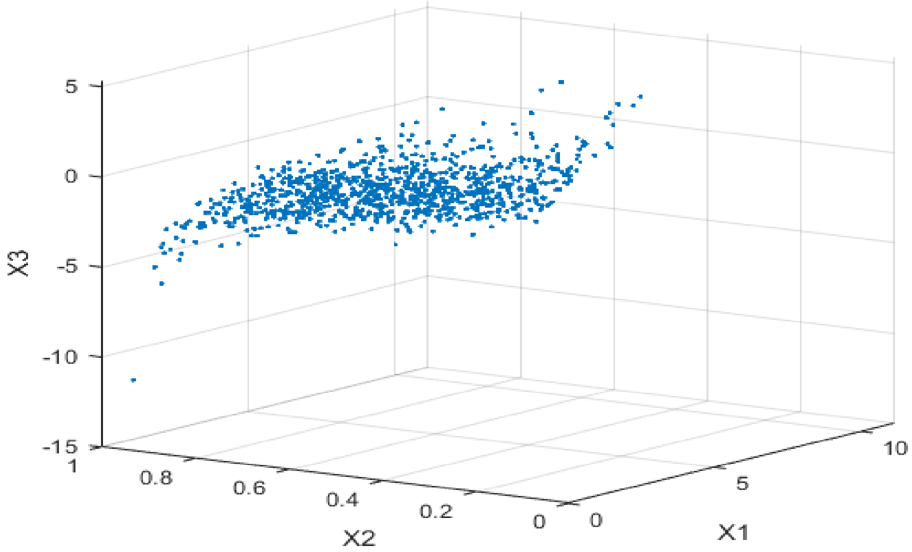

Fig. 1: Three-dimensional Gaussian copula scatter plot of probability density functions

Three-dimensional Gaussian copula scatter plot of pdf (probability density functions)

The three-dimensional Gaussian copula scatter plot of 1000 samples, has a probability density function that does not show any significant upper or lower tail dependence, but it has a moderate relationship between the three axes as can be seen from the cluster, it also has a positive correlation and a non- linear trend, although there are more clusters at the centre as shown in Fig 1. From the simulated plot of 1000 samples, between variable 3 and variable 1, shows that there is a strong clustering relation between both variables, they have a positive correlation, they also don't have any upper or lower tail dependency and they both have a non-linear trend as shown in Fig 2. From the simulated plot of 1000 samples, between variable 3 and variable 2, it is obvious that that there is a strong clustering relation between both variables, they have a negative correlation, they also have moderate upper and lower tail dependency and they both have a linear trend as shown in Fig 3.

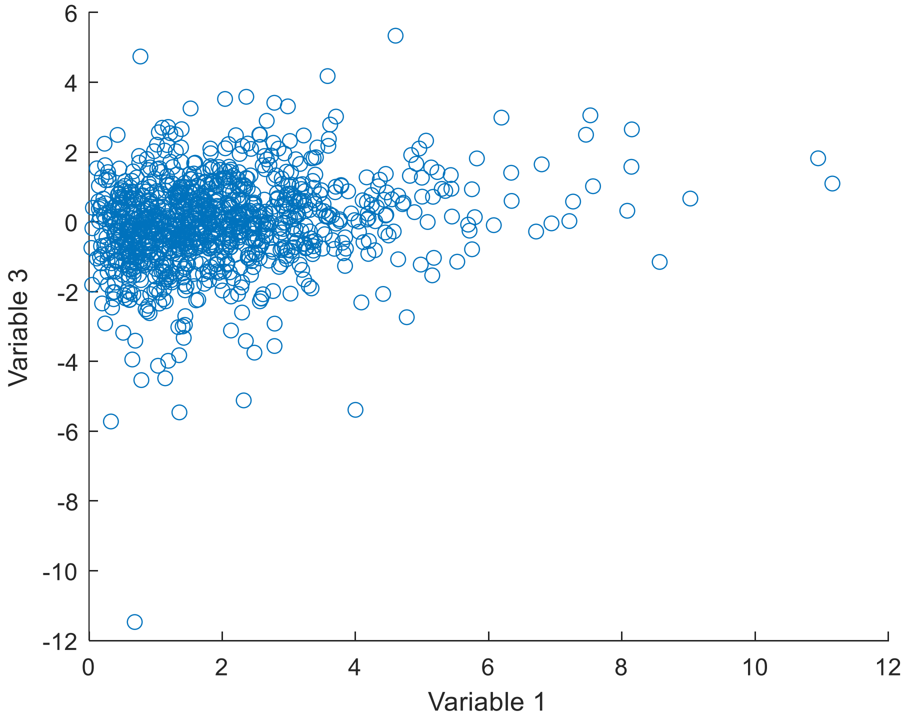

Fig. 2: Wiener plot of variable 3 against variable 1

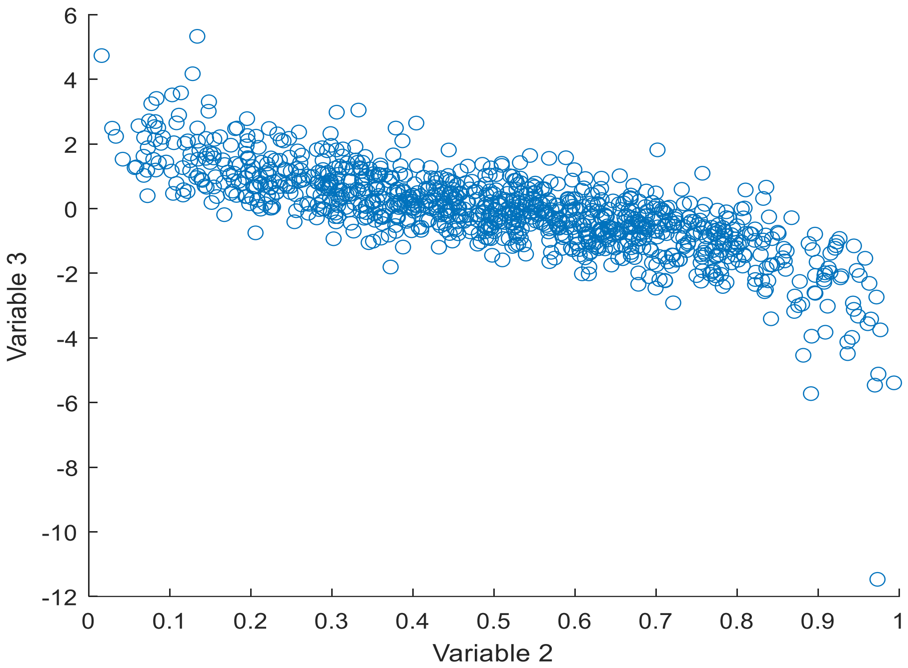

Fig. 3: Wiener plot of variable 3 against variable 2

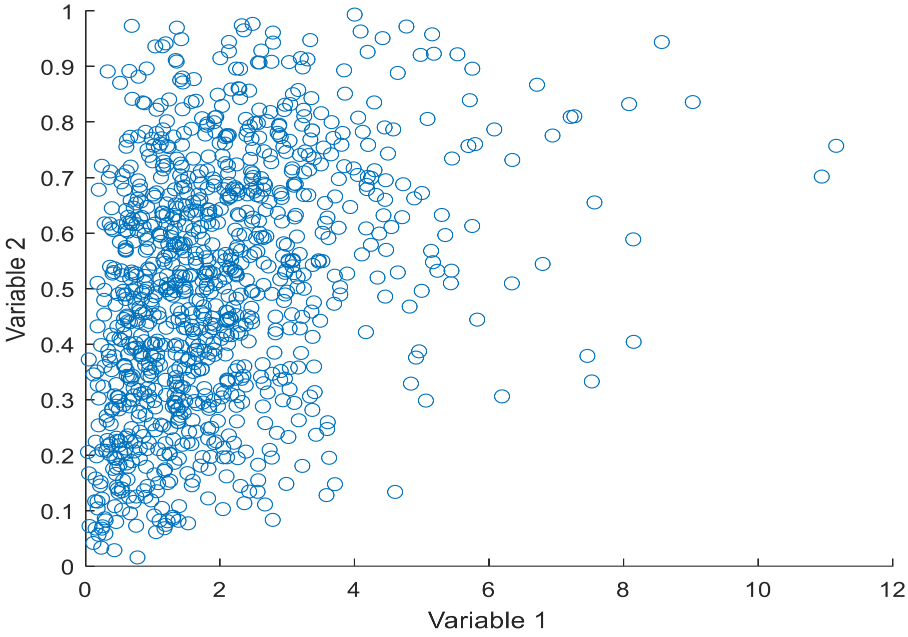

Fig. 4: Wiener plot of variable 2 against variable 1

From the simulated plot of 1000 samples, between variable 2 and variable 1, shows that there is a moderate clustering relationship between both variables, they have a positive correlation, they also have a moderate lower tail dependency and they both have a non-linear trend as shown in Fig 4. From the simulated plot of 100 samples, the matrix scatter plot of the theoretical versus Gaussian rank coefficients shows a positive correlation in column one of row one, while it shows non-correlation in column two and column three of row one. Column one of row two has a non-correlation, while column two of row two has a positive correlation, but column three of row two has a negative correlation. Column one of row three shows a non-correlation, but column two of row three shows a negative correlation while column three of row three has a positive correlation as shown in Fig. 5.

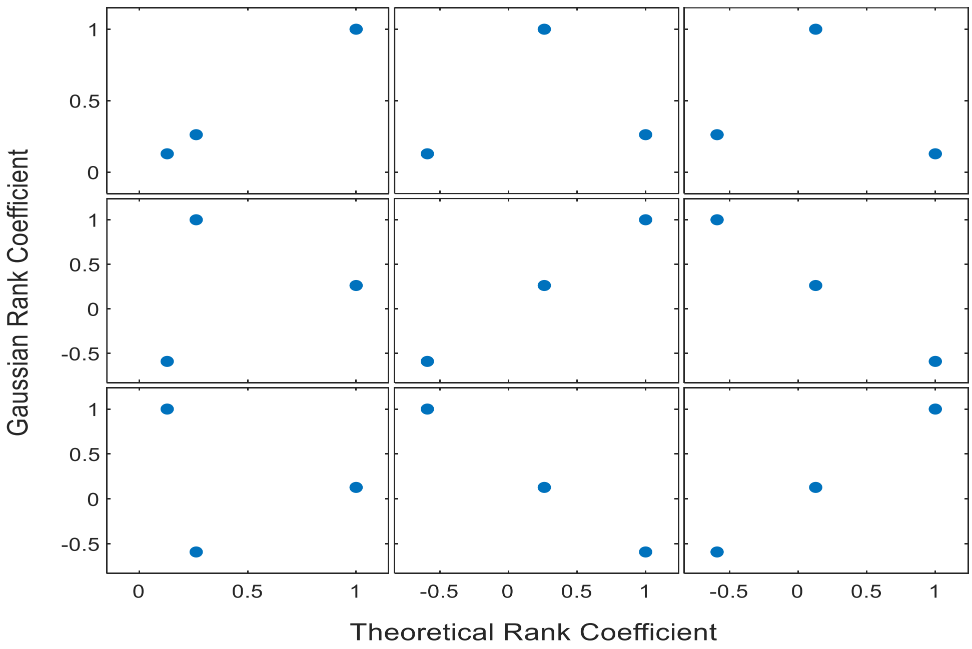

Fig. 5: Matrix plot of theoretical versus Gaussian rank coefficients

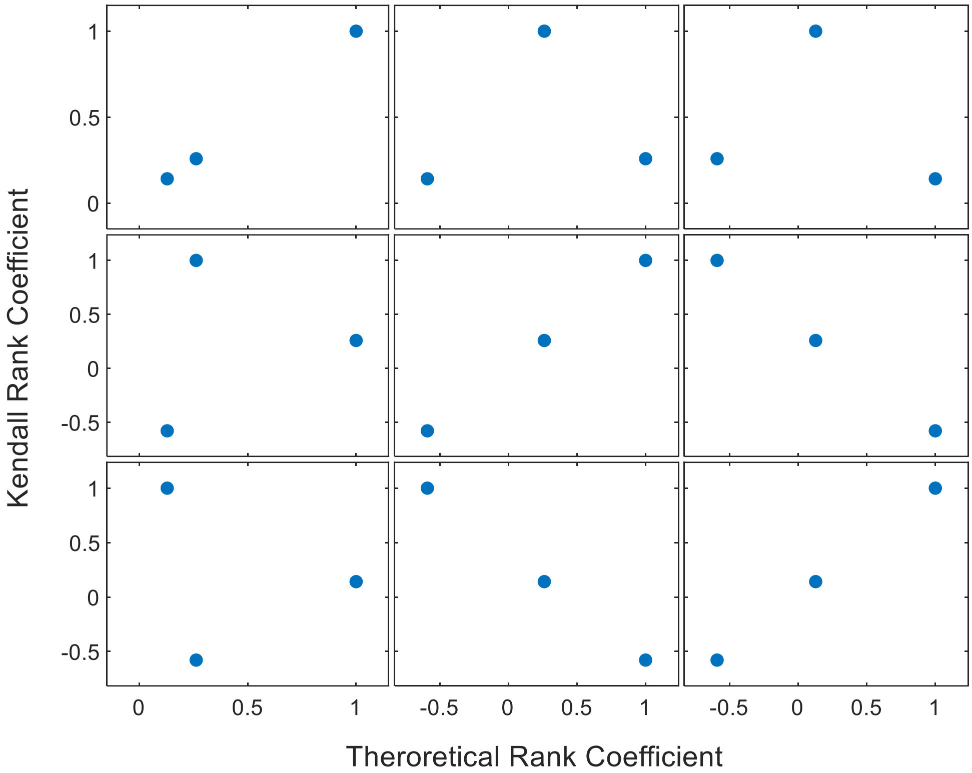

Fig. 6: Matrix plot of theoretical versus Kendall rank coefficients

From the simulated plot of 100 samples, the matrix scatter plot of the theoretical versus Gaussian rank coefficients shows a positive correlation in column one of row one, while it shows non-correlation in column two and column three of row one. Column one of row two has a non-correlation, while column two of row two has a positive correlation, but column three of row two has a negative correlation. Column one of row three shows a non-correlation, but column two of row three shows a negative correlation while column three of row three has a positive correlation as shown in Fig. 6. From Fig 7 it can be seen that the value of pearson dropped from 0.2 to -0.78 and then increased to 1, while the others dropped from approximately 0.18 to approximately -0.6 and then increased to 1 as can be seen in Fig 7. From Fig 8 it can be seen that the value of pearson increased from approximately 0.39 to 1 and then dropped to -0.76, while the others increased from approximately 0.28 to 1 and then dropped to approximately -0.6 as can be seen in Fig 8. From the above plot it can be seen that the value of pearson decreased from 1 to approximately 0.35 and then dropped to approximately -0.22, while the others decreased from approximately 1 to 0.26 and then dropped to approximately -0.13, except for Kendal that dropped to approximately -1.14 as can be seen in Fig 9.

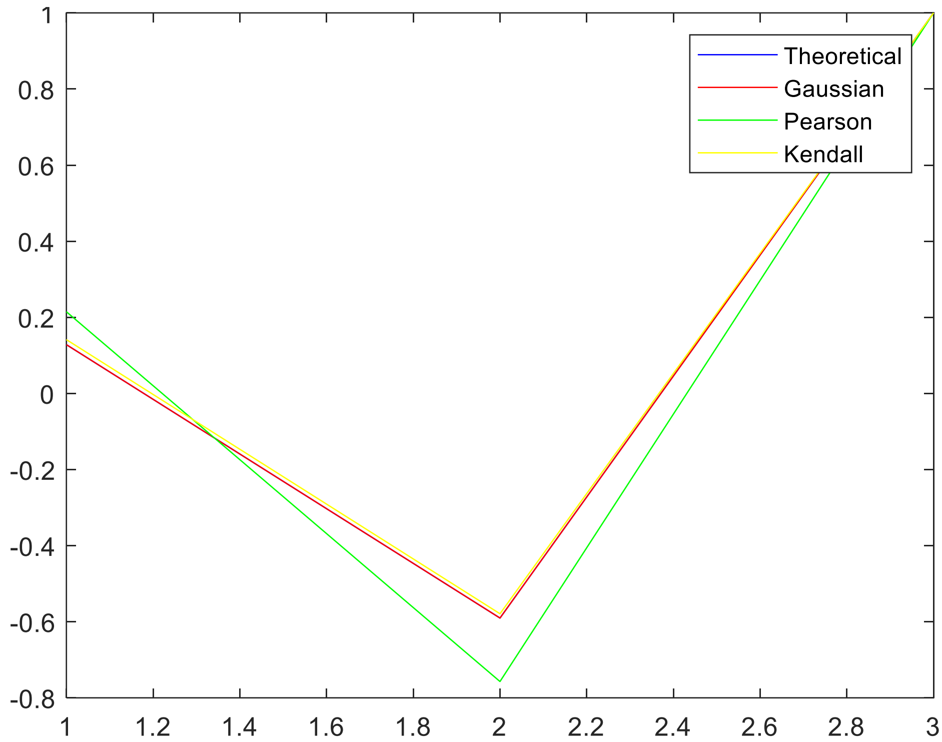

Fig. 7: Comparison of the coefficients of variable 3 after the copular process

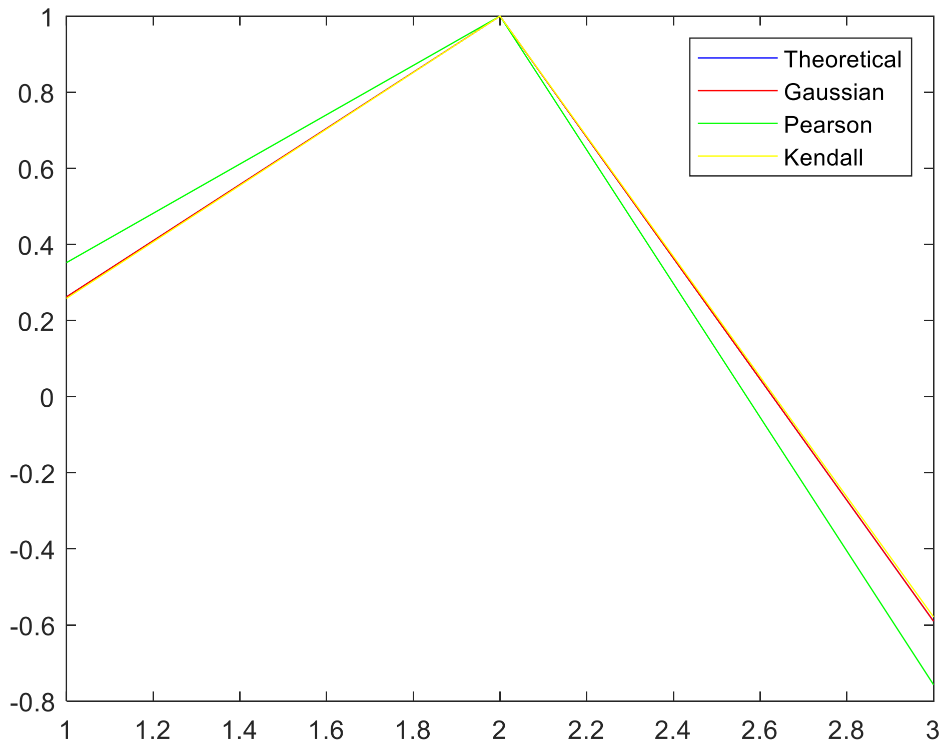

Fig. 8: Comparison of the coefficients of variable 2 after the copular process**.**

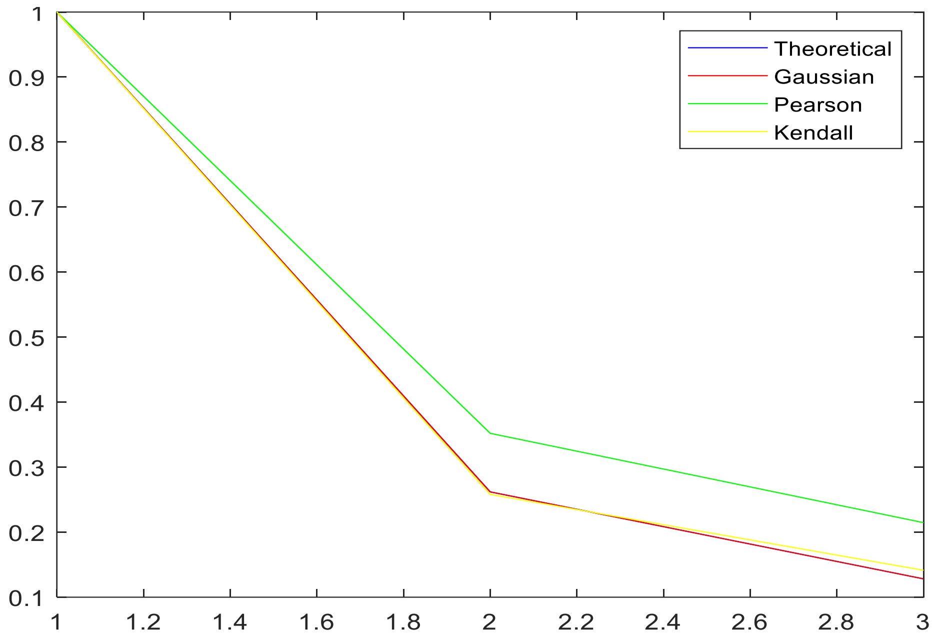

Fig. 9: Comparison of the coefficients of variable 1 after the copular process

4. Conclusion

In this paper, a robust reliability analysis and evaluation method for PLC systems has been proposed. The method which is based on the Wiener process and the Copula function, addresses some of the major shortcomings and limitations of the traditional methods. Results from the evaluation shows that, the Wiener process and the Copula function was effective in the reliability analysis of the PLC and was able to effectively describe the randomness and coupling correlation of the PLC degradation process. The results show that the value of Pearson increased from approximately 0.39 to 1 and then dropped to -0.76, while the scatter plot for the Weiner process has a positive correlation.

The wiener process was used to generate the variable that were applied in the copula function analysis. From the analysis it was observed that the Gaussian rank coefficient and unique deterioration data has identical random variable values as compared to that of the Kendall tau and Pearson correlation coefficient. Also, from the from the three-dimensional Gaussian copula scatter plot of 1000 samples, the probability density function does not show any significant upper or lower tail dependence, but it tends towards a moderate positive dependence.

The study has contributed to the reliability evaluation literature for the PLC system by proposing a hybrid model that has comprehensively determined the Copula function associated with the system. Also, with the proposed model, the copulas used in the study has been accurately described such that how it can be used in the future for studying the dependence or association between random variables has been presented.

References

Bansal, S. and Cheung, S. H. (2017). On the evaluation of multiple failure probability curves in reliability analysis with multiple performance functions. *Reliability Engineering and System Safety*, *167*, 583–594.

Cheng, Y., Zhu, H., Hu, K., Wu, J., Shao, X. and Wang, Y. (2019). Reliability prediction of machinery with multiple degradation characteristics using double-Wiener process and Monte Carlo algorithm. *Mechanical Systems and Signal Processing*, *134*, 106333.

Cherubini, U., Luciano, E. and Vecchiato, W. (2013). Copula Methods in Finance. In *Copula Methods in Finance*.

Di Lascio, F. M. L. and Giannerini, S. (2012). A Copula-Based Algorithm for Discovering Patterns of Dependent Observations. *Journal of Classification*, *29*(1), 50–75.

Gao, H., Cui, L. and Kong, D. (2018). Reliability analysis for a Wiener degradation process model under changing failure thresholds. *Reliability Engineering and System Safety*, *171*, 1–8.

Heping, J., Rui, P., Li, Y., Tianyi, W., Dunnan, L. and Yanbin, L. (2021). Reliability evaluation of demand-based warm standby systems with capacity storage. *Reliability Engineering & System Safety*, *218*.

Bansal, S. and Cheung, S. H. (2017). On the evaluation of multiple failure probability curves in reliability analysis with multiple performance functions. *Reliability Engineering and System Safety*, *167*, 583–594.

Cheng, Y., Zhu, H., Hu, K., Wu, J., Shao, X. and Wang, Y. (2019). Reliability prediction of machinery with multiple degradation characteristics using double-Wiener process and Monte Carlo algorithm. *Mechanical Systems and Signal Processing*, *134*, 106333.

Cherubini, U., Luciano, E. and Vecchiato, W. (2013). Copula Methods in Finance. In *Copula Methods in Finance*.

Di Lascio, F. M. L. and Giannerini, S. (2012). A Copula-Based Algorithm for Discovering Patterns of Dependent Observations. *Journal of Classification*, *29*(1), 50–75.

Gao, H., Cui, L. and Kong, D. (2018). Reliability analysis for a Wiener degradation process model under changing failure thresholds. *Reliability Engineering and System Safety*, *171*, 1–8.

Heping, J., Rui, P., Li, Y., Tianyi, W., Dunnan, L. and Yanbin, L. (2021). Reliability evaluation of demand-based warm standby systems with capacity storage. *Reliability Engineering & System Safety*, *218*.

Hollander, M., and Wolfe, D. A. (1973). *Nonparametric Statistical Methods* (SECOND EDI). wiley.

Joe, H. (1997). *Multivariate Models and Dependence Concepts* (Vol. 73). Chapman & Hall.

Kim, Y. S. and Kolarik, W. J. (1992). Real-time conditional reliability prediction from on-line tool performance data. *International Journal of Production Research*, *30*(8), 1831–1844. https://doi.org/10.1080/00207549208948125

Kruskall, W. H. (1958). ORDINAL MEASURES OF ASSOCIATION. *America Statistical*, *53*(4), 130.

Lehmann, E. L. (1966). Some concepts of dependence. *Annals of Statistics*, *35*(5), 1403–1433.

Lehmann, E. L. (1975). *Nonparametrics: Statistical Methods Based on Ranks.* (Vol. 1). Holden_Day.

Linjie, K., Jianguo, Z., Wang, P. and Qian, W. (2017). Multi-state system reliability analysis of space mechanism in orbit based on performance degradation and universal generating function. *Journal of Mechanical Engineering*, *53*, 20–28.

Nelsen, R. . (2006). An introduction to copulas. In *Springer Series in Statistics* (second). SMC Pneumatics (SEA) Pte Ltd. https://doi.org/10.1515/demo-2017-0006

Pan, D., Wei, Y., Fang, H. and Yang, W. (2018). A reliability estimation approach via Wiener degradation model with measurement errors. *Applied Mathematics and Computation*, *320*, 131–141.

Pan, G., Li, Y., Li, X., Luo, Q., Wang, C. and Hu, X. (2020). A reliability evaluation method for multi-performance degradation products based on the Wiener process and Copula function. *Microelectronics Reliability*, *114*(July), 113758.

Wang, X., Wang, B. X., Jiang, P. H. and Hong, Y. (2020). Accurate reliability inference based on Wiener process with random effects for degradation data. *Reliability Engineering and System Safety*, *193*(August 2019), 106631.

Zhang, J., Ma, X. and Zhao, Y. (2019). Degradation-Based State Reliability Modeling for Components or Systems with Multiple Monitoring Positions. *IEEE/ASME Transactions on Mechatronics*, *24*(6), 2453–2464.